OPTOELECTRONICS AND ADVANCED MATERIALS – RAPID COMMUNICATIONS Vol. 2, No. 12, December 2008, p. 758 – 762

Computer vision measurement new algorithm and uncertainty evaluation

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Usually, engineers use the least square method to solve the equation group in machine vision measurement task. By algebra principle, the least square method is inefficient for linear correlated variable. In this case, big error exists in the measuring result. In some machine vision task, people only want to know the depth information. In order to solve those problems, this paper proposed new method based on the parametric equation originally. Use parametric equation to define the machine vision system model and define scene points as quaternion. We can calculate the depth information by decomposing the equation group. And evaluation of variance is easier by this equation. Meanwhile, in order to illustrate the variance this paper use Hough transform to the equations. The line in the original coordinate system will change to a point in another coordinate system. Many points fit a line to denote a point in the original coordinate system. The experiment in the end proves the algorithm effective.

(Received November 5, 2008, accepted November 27, 2008)

Keywords: Machine vision measurement, Three-dimensional reconstruction, Error analysis, Uncertainty

1. Introduction

In some projects, engineer usually wants to know the depth information or shape of the observed object. Machine vision is a good choice for some non-contact measurement task. Machine vision measurement is a method that reconstruct three-dimensional object by at least two photos taken by camera system. The available algorithm is based on collinear equation known by projection matrix. We can use many equations to calculate the value of the three-dimensional point [1,2]. The engineers not only want to know the measurement value, but also concern with the accuracy and the error factors. Ma and Zhang [3] give us a way to evaluate the accuracy that based on error transmission formula. But this way is difficult and complicated, especially for multi-camera system. Zhou [4] uses the project matrix to evaluate the uncertainty directly, but it is complicated too. Even if we only want to know the scene point z-coordinate value, we have to calculate all three dimensional coordinates. There have the same question for the method based on Richard Hartley's eight points [5]. Besides, some engineers use the grey theory to resolve the problem [6,7].

This paper analyzes the principle of machine vision deeply, and proposes a new algorithm. This algorithm is based on the parametric equation. In machine vision model, the rays are denoted by parametric equation. According to the algebra principle, a line can be defined as a three equations. Each equation only has the relationship with one basic unit of the coordinate system. When we only want to know the z-coordinate information, we can calculate the equation that has relationship with z unit. By this method, the uncertainty evaluation is easier. Meanwhile, usually we use the least square method evaluate the answer. If the variable is linear correlated, the least square method is inefficient. Through this method the correlation could be avoid.

In order to illustrate the variance in the machine vision task, this paper use Hough transform to the equations. The line in the original coordinate system will change to a point in another coordinate system. One point means one camera. Many points fit a line to denote a point in the original coordinate system.

2. Machine vision principle

The goal of a machine vision system is to create a model of the real world from images. A machine vision system recovers useful information about a scene from its two-dimensional projections. The principle is showed by Fig. 1. We take the photos of scene point in the World coordinate. When we know the world coordinate system, we would certain the camera coordinate value. In the figure1, the location of P can be certain by the intersection of line $s_1 p$ and $s_2 p$. We will know the internal and external parameters of the camera system after calibrating the system. So we could know the project matrix M1and M2. From the reference [2], we can get the following equations:

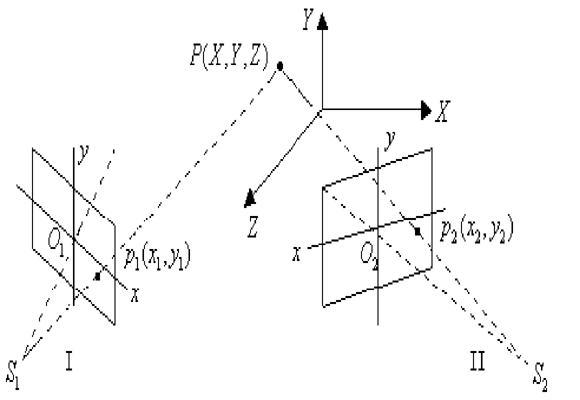

Fig. 1. Computer vision model.

$$Z_{ci}\begin{bmatrix}u_i\\ v_i\\ 1\end{bmatrix}=\begin{bmatrix}m_{11}^i & m_{12}^i & m_{13}^i & m_{14}^i\\ m_{21}^i & m_{22}^i & m_{23}^i & m_{24}^i\\ m_{31}^i & m_{32}^i & m_{33}^i & m_{34}^i\end{bmatrix}\begin{bmatrix}X\\ Y\\ Z\\ 1\end{bmatrix} \qquad (1)$$

We could get (X,Y,Z) from equation (1). In fact, multi-camera system would be used. Meanwhile, those equations are incompatible because of the error existence. People use the Least Square Method to solve the problem. In practice, the error all exists. We rewrite (1) as: $AP=b$, here, $P=\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}$. According to the least square method,

$$P=(A^TA)^{-1}A^Tb \qquad (2)$$

But here have three questions:

(1) By principle, the least square method is inefficient for linear correlated variable. In this case, big error exists in the measuring result. If X,Y,Z have some extent linear correlation, the inverse of matrix (A^TA) cannot be calculated or almost is equal to zero.
So the result is unstable.
(2) The variance evaluation is very complicated.
(3) In some task, the engineer only concern with the depth information, but they have to calculate all coordinates.

3. The new algorithm

The machine vision system model is showed by Fig. 2. Assume that we have known the internal and external parameters. We know from the Fig. 2 that $\overrightarrow{O_0P}=\overrightarrow{O_0O_1}+\overrightarrow{O_1P}$. The superscripts express different coordinates. So ${}^{C}\overrightarrow{O_0O_1}=(0,0,f)$ in camera coordination system and ${}^{C}\overrightarrow{O_1P}=(x,y,0)$. Here, the focal length is denoted by f. The image coordinates are denoted by x,y.

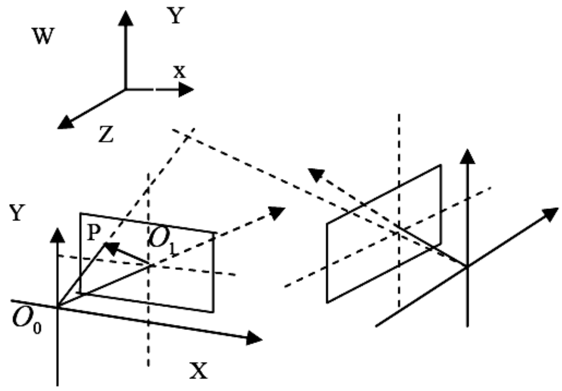

Fig. 2. The principle of algorithm.

The pixel value is denoted by u,v.

$$\overrightarrow{O_0P}=\begin{bmatrix}x\\ y\\ f\end{bmatrix}=\begin{bmatrix}dx & 0 & -u_0dx\\ 0 & dy & -v_0dy\\ 0 & 0 & f\end{bmatrix}\begin{bmatrix}u\\ v\\ 1\end{bmatrix} \qquad (3)$$

Here, symbol is same with the reference [3]. Assume that we have calibrated the camera, then

$${}^{w}\overrightarrow{O_0P}=\begin{bmatrix}x\\ y\\ f\end{bmatrix}=\begin{bmatrix}r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33}\end{bmatrix}\begin{bmatrix}dx & 0 & -u_0dx\\ 0 & dy & -v_0dy\\ 0 & 0 & f\end{bmatrix}\begin{bmatrix}u\\ v\\ 1\end{bmatrix} \qquad (4)$$

From (4), we could get the equation of the ray line in world coordination system.

Let

$$\begin{bmatrix}x\\ y\\ f\end{bmatrix}=\begin{bmatrix}m\\ n\\ p\end{bmatrix},$$

So

$${}^{w}\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}=\begin{bmatrix}m\\ n\\ p\end{bmatrix}_i * k_i+\begin{bmatrix}t_1\\ t_2\\ t_3\end{bmatrix}_i \qquad (5)$$

Here, k_i is the proportion coefficient. (t_1,t_2,t_3) is the movement vector and i is camera. We will get tree equations with two unknown factor by equation group (5). From equation (5), we can describe the scene point as quaternion ${}^{w}(x, y, z, k)$ .In order to calculate ${}^{w}(x, y, z)$, we need know the constraint relationship a mong k_i. When we have i cameras, we will have $3+i$ unknown factors and $3\times i$ equations. So we can constraint those k_i. When we have two or more cameras, we can get ${}^{w}(x, y, z)$. Usually, we use the Least Square Method to solve the equation group because of error existence.

$$AP = b \tag{6}$$

$A = \begin{bmatrix} 1 & m_1 \\ 1 & m_2 \\ \cdots & \cdots \\ 1 & m_n \end{bmatrix}$, here, m_i is the one element of the vector ${}^{W}\overrightarrow{O_0P} = (m, n, p)$, $P = \begin{bmatrix} x \\ l \end{bmatrix}$, x is the one basic unit of the three-dimensional coordinate system. l is the distance. $b = \begin{bmatrix} t_1 \\ t_2 \\ \cdots \\ t_3 \end{bmatrix}$ Here, t_i mean the item of the movement vector.

So

$$P = (A^T A)^{-1} A^T b \tag{7}$$

From equation (5), when we want to calculate the z-coordinate, we don't need to evaluate the x coordinate and y-coordinate. It is easier to calculate z-coordinate. Meanwhile we know from (6), if X, Y, Z have some extent linear correlation, the inverse of matrix $(A^T A)$ can not be effect. Because the matrix A only has relationship with m_i, here m_i is the one element of the vector.

4. Uncertainty analysis

In fact, the errors would corrupt accuracy. In order to evaluate and illustrate the measurement variance, we transform equation (5) by Hough transform. Fig. 3 illustrates the principle.

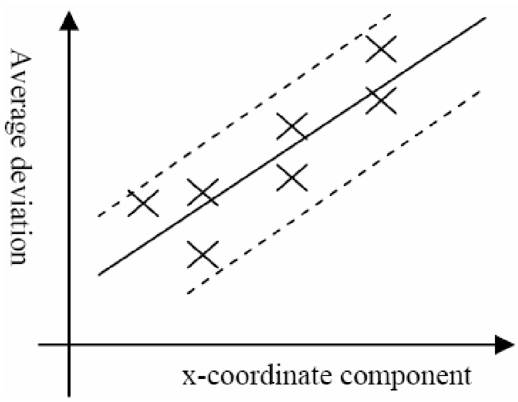

Fig. 3. Figure after transform.

X-coordinate is one element of the movement vector; the y-coordinate is the element of the vector ${}^{W}\overrightarrow{O_0P}$. Each cross expresses a ray of the original coordinate system. There are seven cameras in the figure. From this figure we can know the deviation. Use the Least Square Method to fit the line equation. Set $y_i = t_i$, here i is camera. a_0, a_1 is line parameters that we need to fit. x_i is one element of vector ${}^{W}\overrightarrow{O_0P} = (m, n, p)$. So according to the Least Square Method [4,5] the weighted square of deviation should be minimal.

$$\sum_{i=1}^{N} [y_i - (a_0 + a_1 x_i)]^2 |_{a=\hat{a}} \tag{8}$$

That means the optimal estimation for parameter a (represent a_0 , a_1), that need the weighted square of deviation minimal.

$$\frac{\partial}{\partial a_0} \sum_{i=1}^{N} [y_i - (a_0 + a_1 x_i)]^2 |_{a=\hat{a}}$$
$$= -2 \sum_{i=1}^{N} (y_i - \hat{a}_0 - \hat{a}_1 x_i) = 0,$$
$$\frac{\partial}{\partial a_1} \sum_{i=1}^{N} [y_i - (a_0 + a_1 x_i)]^2 |_{a=\hat{a}}$$
$$= -2 \sum_{i=1}^{N} (y_i - \hat{a}_0 - \hat{a}_1 x_i) = 0.$$

Then we can know

$$\begin{cases} \hat{a}_0 N + \hat{a}_1 \sum x_i = \sum y_i, \\ \hat{a}_0 \sum x_i + \hat{a}_1 \sum x_i^2 = \sum x_i y_i. \end{cases} \tag{9}$$

From the equation group (9) to get the optimal estimation,

$$\hat{a}_0 = \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2}$$
$$\hat{a}_1 = \frac{N\left(\sum x_i y_i\right) - \left(\sum x_i\right)\left(\sum y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} \tag{10}$$

The uncertainty is not zero because of the error existence. We use the standard deviation S to represent the uncertainty.

$$x_{\min}^2 = \frac{1}{S^2} \sum_{i=1}^{N} [y_i - (\hat{a}_0 + \hat{a}_1 x)]^2. \tag{11}$$

$$S = \sqrt{\frac{1}{N-2} \sum_{i=1}^{N} [y_i - (\hat{a}_0 + \hat{a}_1 x_i)]^2}. \tag{12}$$

5. Simulation

In order to control the experiment error, we use matlab7.0 to simulate the original algorithm and the algorithm proposed by this paper. We add error with different variance to the data. The simulation would prove the validity of the algorithm.

Set the point coordinate (2, 1, 3) in the world coordinate system, and the camera focal $f = 3cm$. Set pixel $d_x = d_y = 0.05cm$, the CCD center $(u_0, v_0) = (5,4)cm$. We use six cameras system. Define rotation of x, y, z-coordinates as $r = (\alpha, \beta, \gamma)$ and translation vector t as follows:

$r_1 = (pi/12, pi/12, pi/20), t_1 = (110,120,200);$

$r_2 = (pi/12, pi/12, pi/10), t_2 = (120,120,100);$

$r_3 = (pi/12, pi/10, pi/20), t_3 = (120,150,200);$

$r_4 = (pi/10, pi/12, pi/20), t_4 = (120,120,150);$

$r_5 = (pi/12, pi/5, pi/20), t_5 = (130,120,200);$

$r_6 = (pi/12, pi/15, pi/20), t_6 = (120,140,100);$

Add random error to the measurement datum. The STD σ is span from 0 to 1 and the step is 0.1.

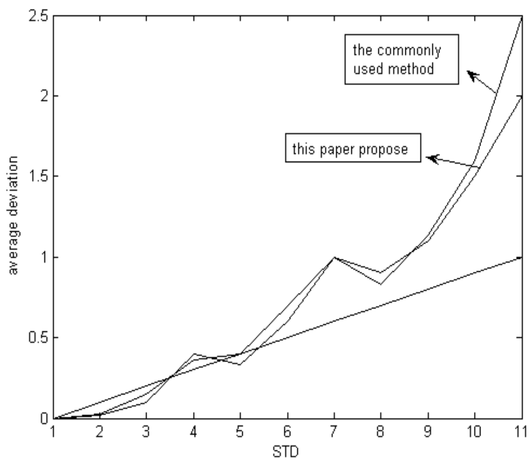

Fig.4.The deviation of different STD.

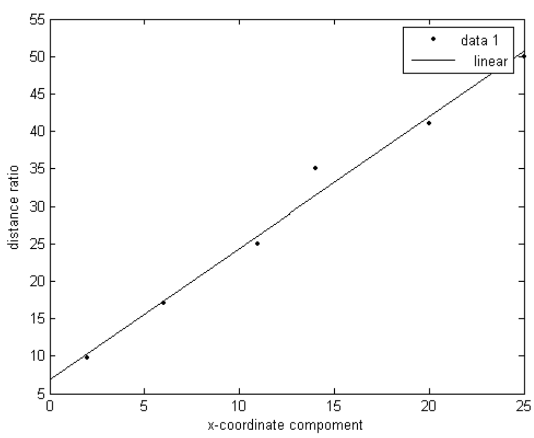

Fig.5. X-coordinate component fitting.

The average deviation is drawn in figure 4. We can know, two methods are almost same. When we only want to know x-coordinate and variance, we don't need to calculate all coordinates. But the original algorithm require calculate x coordinates, y-coordinate, z-coordinate, then evaluate the variance. That means the original algorithm will corrupt the precision.

Example: detect the corner point and evaluate the uncertainty

The observed object is a chessboard plane. Use the machine vision system to measure the corner point. We can get the corner point by detecting algorithm. We stable the device on the optical table after calibrating the camera. The system showed by following figure. The image data is send to computer by wire.

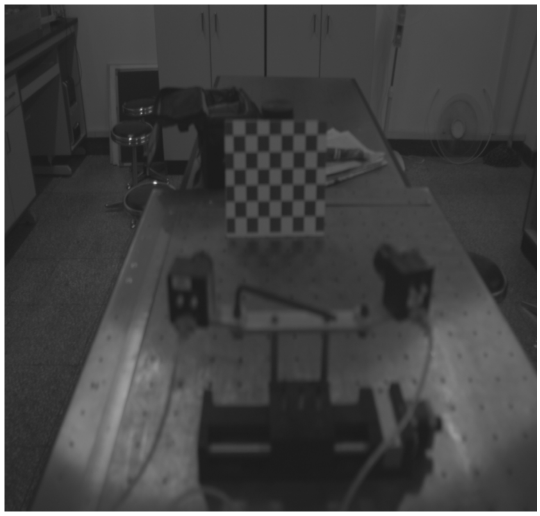

Fig.6. Experiment device.

We calculate the coordinate of the corner point by the original algorithm and the algorithm propose by this paper. The target point is the second corner on the left in the chessboard plane.

Real value (4.8, 6.4, 0) (cm)	The present algorithm	The paper proposed algorithm	X STD
1	(5.12, 6.63, 0.14)	(5.12, 6.63, 0.13)	no
2	(5.09, 6.33, -0.20)	(5.09, 6.33, -0.20)	no
3	(4.80, 6.41, 0)	(4.80, 6.41, 0)	no
4	(4.91, 6.27, 0.13)	(4.91, 6.37, 0.13)	no
5	(4.7, 6.15, 0.20)	(4.7, 6.25, 0.21)	no
6	(4.75, 6.30, 0.10)	(4.77, 6.35, 0.14)	0.28
7	(4.74, 6.30, 0)	(4.78, 6.33, 0)	0.31
8	(4.70, 6.20, -0.10)	(4.75, 6.70, -0.10)	0.34
9	(4.92, 6.30, 0.10)	(4.85, 6.60, 0)	0.29
10	(4.78, 6.50, 0)	(4.83, 6.50, 0)	0.27

The first five data is gotten by two-camera system, the last five data is gotten by the optimum of multi-camera system images. The result is showed by the following figure. The result of original algorithm is denoted by asterisk.

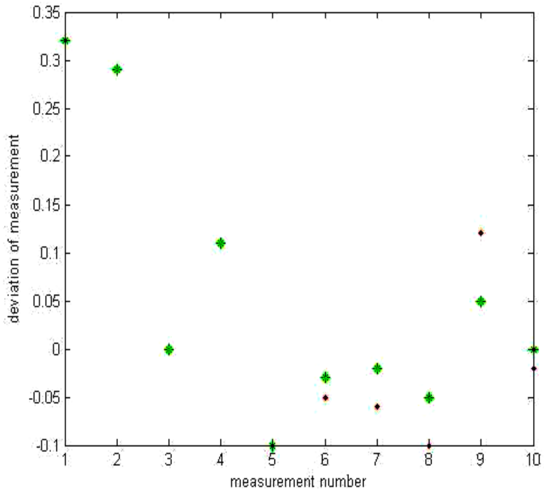

Fig.7. The compassion of measurement of x coordination.

From the figure, we can know the different of result of two algorithms is small for two camera system. But for multi-camera system is different. This paper proposed algorithm is easy. And we can evaluate the variance easily. The essence of this paper proposed algorithm is parameter equation.

6. Discussion

Known from the experiment result, the essence of the original algorithm and the paper proposed is almost same. But this paper proposed algorithm could evaluate deviation of one basic unit of three dimension coordinate system easily. When we only want to calculate the depth information, this algorithm is better.
For example, if we only want to calculate the z-coordinate, we don't need to calculate x-coordinate and y-coordinate. And the variance calculation is easier than original algorithm. Meanwhile, this algorithm is stable.

7. Summary

In order to solve the stability of machine vision system and be easier to evaluate the variance, this paper proposed a new algorithm for machine vision system. When engineer only want to know the depth information, this algorithm is more reasonable. The experiment in the end proved the algorithm effective.

Acknowledgements

This project is supported by National Natural Science Foundation of China (50675015), Funding Project for Academic Human Resources Development in Institutions of Higher Learning under the Jurisdiction of Beijing Municipality(PXM2007-014224-044655).

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