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Chaotic ferroresonance and its control with sliding mode technique for voltage transformer circuits: a case study of manual single phase switching operation in three-phase transmission system

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Chaotic ferroresonance is a nonlinear dynamic electrical phenomenon, which frequently occurs in a power system that comprises no-load saturable transformers, transmission lines (or cables) and single-phase switching with three-phase supply. This paper presents an extension of chaotic nonlinear bifurcation analysis on ferroresonance in a case study of manual single phase switching operation in three-phase transmission system. Analysis and classification methods are presented which provide chaotic analysis insights into the global behavior of ferroresonance. Analytical methods for nonlinear dynamic systems and MATCONT are used to exhibit characteristic curves such as a time-domain waveform, phase-plane, and bifurcation points.

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1. Introduction

Ferroresonance is due to the interaction between a nonlinear inductance and a capacitance. The nonlinear inductance is typically the saturable magnetizing inductance of a transformer, whereas the capacitance can be ascribed to distribution cables, transmission lines, capacitor banks, voltage grading capacitors in HV circuit breakers or by the coupling between double circuit lines [1]. Ferroresonance is initiated by improper switching operation, routine switching, or load shedding involving a high voltage transmission line. It can result in unpredictable over voltages and high currents. The energization and deenergization by manual single phase fuse cutout switching operation or by abnormal situation (unbalanced faults) in three-phase transmission systems, consisting of a series/parallel combination of an unloaded or very light loaded transformer with saturation characteristic and capacitor in the form of transmission-line capacitive coupling, present high potential for the occurrence of ferroresonance [2].

Ferroresonance is a jump resonance, which can suddenly jump from one normal steady-state response (sinusoidal line frequency) to another ferroresonance steady-state response. It is characterized by a high overvoltage and random time duration, which can cause dielectric and thermal problems to the transmission and distribution systems and switchgear. Typical cases of ferroresonance are reported in Refs. [3] and [4]. Theory of nonlinear dynamics has been found to provide deeper insight into the phenomenon. References [5] to [7] are among the early investigations in applying theory of bifurcation and chaos to ferroresonance. The susceptibility of a ferroresonant circuit to a quasi-periodic and frequency locked oscillations are presented and, the effect of initial conditions is investigated in references [8] and [9]. Ref. [10] is a milestone contribution highlighting the effect of transformer modeling on the predicted ferroresonance oscillations. The present paper addresses the effect of nonlinear core on the global behavior of a ferroresonant circuit.

2. Ferroresonance and chaotic behaviour

Nonlinear dynamical systems can exhibit multiple equilibrium points (the point that the system operates without change), limit cycle, jump resonance and subharmonic generation. Such systems show a high sensitivity to initial conditions, which determine which steady state mode will result. Steady state responses can be either periodic or chaotic (nonperiodic). In ferroresonance situation, remnant magnetization of the cores, voltage at the time of manual switching, and amount of charge on the capacitance are initial conditions which determine the steady state response. Even small changes in such initial conditions, it is possible that subsequent initiations of ferroresonance may result in very destructive voltage waveforms.

The theories of nonlinear dynamics and chaos can now be used to help analyze, model, understand ferroresonance phenomenon. Simplified methods of descriptively categorizing periodic and chaotic modes of ferroresonance will be presented here. Phase plane diagrams are used to distinguish between various periodic modes of ferroresonance.

The connection of ferroresonance to nonlinear dynamics and chaos was established in 1988 and published in 1992 [11].Nonlinear dynamics was recently applied to ferroresonance [12], but that paper did not address the connection to chaotic systems. Gleick [13] provides an excellent conceptual introduction to nonlinear dynamics and chaos. Baker and Gollub [14], Thompson and Stewart [15] and Schuster [16] provide a good theoretical introduction for chaotic ferroresonance phenomenon into power engineering literature.

In this study, major analytical tools are used to exhibit and analyze nonlinear ferroresonant system are as follows:

a. Bifurcation diagram

The bifurcation diagrams allow a comparison between the periodic and chaotic behaviors of the system. As the operating condition (for example the magnitude of the supply voltage) of a nonlinear system changes, the equilibrium point can change along the number of equilibrium points. The values of these parameters, which start to produce different steady-state conditions, are known as critical or bifurcation values. A bifurcation diagram is a plot that displays single or multiple solutions (bifurcations) as the value of the control parameter is increased.

b. Phase plane

A phase plane is a visual display of certain characteristics of certain kinds of differential equations; it is a 2-dimensional version of the general n-dimensional phase space. Phase planes are useful in visualizing the behavior of physical systems. The phase plane analysis is a graphical method, in which the time behavior of a system is represented by the movement of a point representing the state variables of the system with time. As time evolves, the initial point follows a trajectory. If a trajectory closes on itself, then the system produces a periodic solution. In the chaotic system, the trajectory will never close to become cycles. This tool is useful in determining if the dynamics are stable or not.

3. System modelling

Equivalent circuit of power transformer in three phase transmission system is shown on Fig. 1. More practical representations are also described into the literature. As in given figure below, the nonlinear inductances represent iron core coils of unloaded three phase power transformer and the capacitance (C_A, C_B, C_C) represents capacitance of long transmission line (or underground cable). In this case, the ferroresonance occurs because of the malfunction of one pole of circuit breaker.

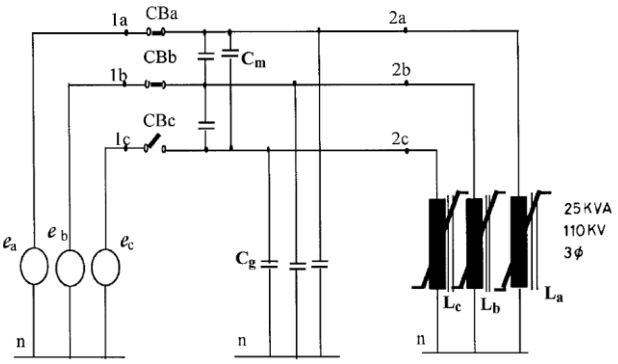

Fig. 1. Single phase switching ferroresonance in three phase voltage transformer.

After single-phase switching-on of the third phase of an unloaded transmission transformer two identical series circuits arise each consisting of the no-load inductance of the transformer and the earth capacitance of the still open phases.

To obtain the equivalent circuit of the system, we define nonlinear iron cored inductance on the single phase open transformer as a resistor which resembles the transformer losses and a parallel connected nonlinear inductance. It was shown in Fig. 2.

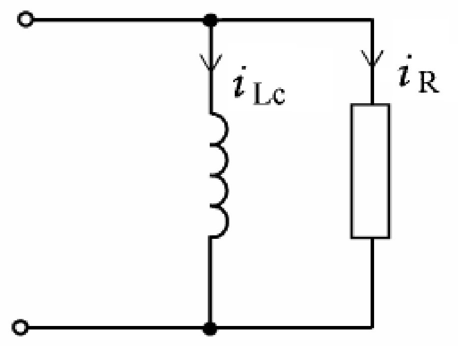

Fig. 2. Nonlinear inductance model.

Then getting thevenin equivalent circuit of single open phase Fig. 3 can be obtained.

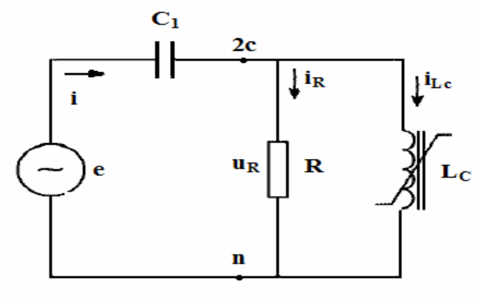

Fig. 3. Basic ferroresonance circuit.

By Kirchhoff's current law;

$$-i + i_R + i_{Lc} = 0 \tag{1}$$

Due to the nonlinear characteristics of the transformer, current would be defined as 11th-order polynomial equation of flux parameter [7xx].

$$i_L(\phi) = a\phi + b\phi^{11} \tag{2}$$

Combining (1) and (2), following equation is obtained.

$$C_1 \frac{d(u_R - e)}{dt} + \frac{u_R}{R} + a\phi + b\phi^{11} = 0 \tag{3}$$

If u_R voltage expression in the equation is written in flux, a second order differential equation is obtained.

$$u_R = \frac{d\phi}{dt} \tag{4}$$

$$\frac{d^2\phi}{dt^2} + \frac{1}{RC_1}\frac{d\phi}{dt} + \frac{1}{C_1}(a\phi + b\phi^{11}) = \omega E_m \cos(\omega t) \tag{5}$$

Capacitance value C_1 of the circuit refers phase-neutral and phase-phase capacity values in equation.

$$C_1 = C_g + 2C_m \tag{6}$$

Three-phase power circuit shown in Fig. 1 comprises 100MVA and 110/44/4 kV label power transformer. Different losses occurring in transformer substations in the system that may play an important role in determining ferroresonance mode. In case of different line lengths and transformer losses, possible ferroresonance events occur in the system have been investigated. Calculations have been handled not on the actual values of the system parameters in the system, but per-unit values.

$$V_{base} = \frac{110}{\sqrt{3}} kV,$$

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{(63{,}5 \times 10^3)^2}{100 \times 10^6} = 40{,}33\Omega$$

$$\omega_{base} = 2\pi 50 rad/s$$

Investigated length of transmission line is 62 km. Capacitance values are taken as for Cg = 5.41 nF and Cm = 1.18 nF per kilometer. In case of selection of the transmission line is 62km, circuit capacitances are calculated as Cg = 338,125 nF and Cm = 73.75 nF. Capacitance value of C1 in the second order differential equation becomes as follows:

$$C_1 = 338.125 + 2 \times (73.75) = 485.625 nF$$

R-transformer losses values were taken in four different values in the system as R_1=1008Ω, R_2=4033Ω, R_3=6048Ω. Source voltage (E) and angular velocity (w) were fixed 1 pu values. Transformer losses and the capacitance in the circuit were translated into per-unit expressions;

$$R_{1p.u} = \frac{1008}{40.33} = 25.00 p.u$$

$$R_{2p.u} = \frac{4033}{40.33} = 100.00 p.u$$

$$R_{3p.u} = \frac{6048}{40.33} = 150.00 p.u$$

$$X_c = \frac{1}{\omega C} = \frac{1}{314 \times 485.625} = 6563.09\Omega$$

$$X_{c_{p.u}} = \frac{6563.09}{40.33} = 162.73 p.u$$

$$C_{p.u} = \frac{1}{\omega_{p.u} \times X_{c_{p.u}}} = \frac{1}{1 \times 162.73} = 0.00614 p.u$$

After the translation of the circuit parameters into per-unit values, need for variable transformation on the second-order differential equation is required. Parameter a and b in nonlinear current equation are taken as 2.8 × 10^{-3} and 7.2 × 10^{-3} respectively. State variables of system were extracted as follows;

$$\phi = x_1 \qquad \frac{d\phi}{dt} = x_2 = \dot{x}_1 \qquad \frac{d^2\phi}{dt^2} = \dot{x}_2 \qquad x_3 = \omega t$$

$$\dot{x}_2 = -\frac{1}{RC_1}x_2 - \frac{1}{C_1}(ax_1 + bx_1^{11}) + \omega E_m \cos(x_3)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\dfrac{1}{RC_1}x_2 - \dfrac{1}{C_1}(ax_1 + bx_1^{11}) + \omega E\cos(x_3) \\ \omega \end{bmatrix}$$

4. Simulation results and discussion

State equations were modeled into Matlab Simulink environment as shown in Fig. 4.

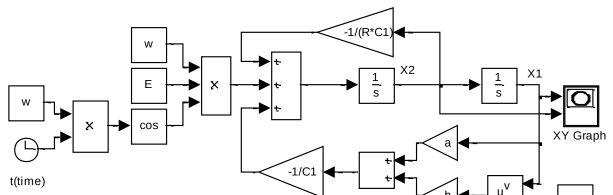

Fig. 4. Matlab simulink mathematical model of ferroresonance circuit.

Simulation results for R=25 pu have showed in Fig. 5-6-7. Initial conditions are taken as x1=0, x2=0, t=0.

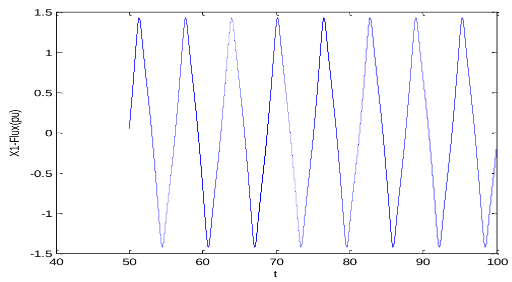

Fig. 5. Flux (x1 state variable) change over time for R=25 pu.

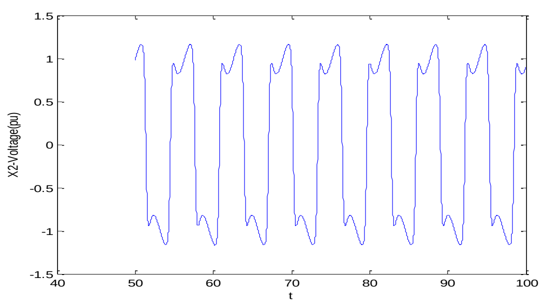

Fig. 6. Voltage (x2 state variable) change over time for R=25 pu.

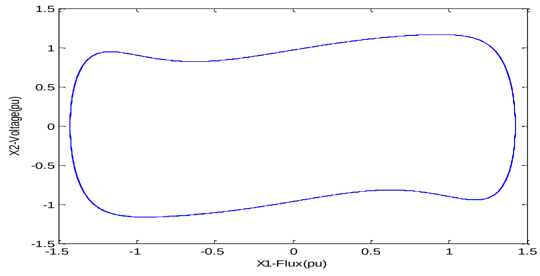

Fig. 7. Voltage–flux (x1-x2) phase portrait for R=25 pu.

In these three figures basic ferroresonance situation was captured. Ferroresonant behaviour was demonstrated by the distortion and high amplitude of transformer voltage waveform and flux as shown in Fig. 5 and Fig. 6.

The phase plane diagram of Fig. 7 shows the characteristics of a periodic waveform with a frequency equal to the system frequency.

Simulation results for R=100 pu have showed in Fig. 8-9-10. Initial conditions are taken as x1=0, x2=0, t=0. Subharmonic ferroresonance is demonstrated in Fig. 8 and Fig. 9 showing transformer voltage waveform and flux for R=100 pu.

For this type of ferroresonance, the phase plane diagram of Fig. 10 shows separate trajectories closing on themselves.

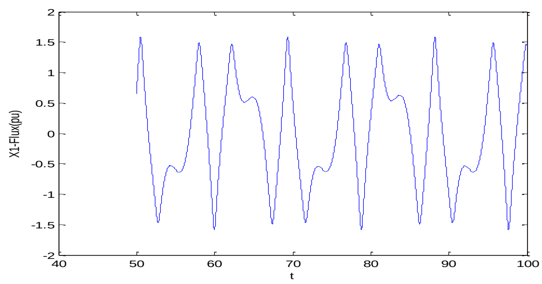

Fig. 8. Flux (x1 state variable) change over time for R=100 pu.

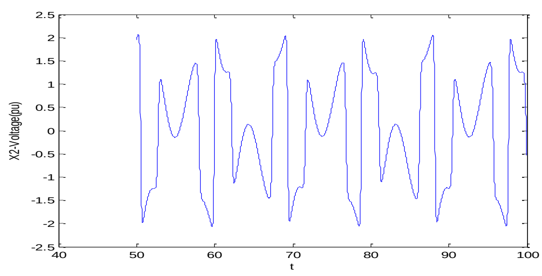

Fig. 9. Voltage (x2 state variable) change over time for R=100 pu.

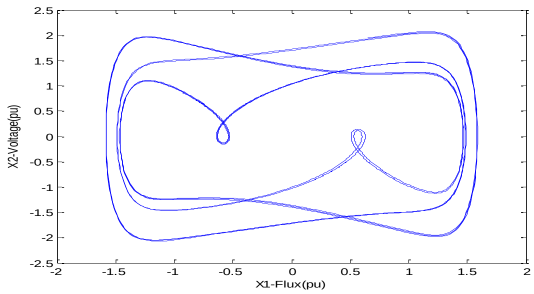

Fig. 10. Voltage–flux (x1-x2) phase portrait for R=100 pu.

Simulation results for R=150 pu have showed in Fig. 11-12-13. Initial conditions are taken as x1=0, x2=0, t=0. Chaotic ferroresonance is demonstrated in Fig. 11 and Fig. 12 showing transformer voltage waveform and flux for R=150 pu.

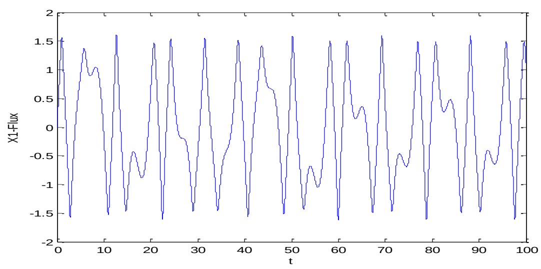

Fig. 11. Flux (x1 state variable) change over time for R=150 pu.

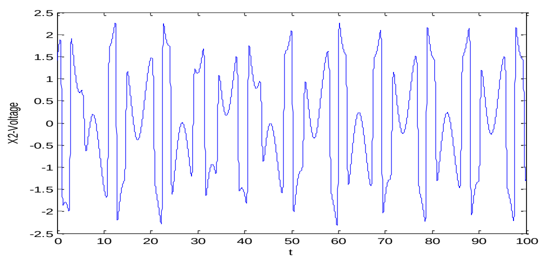

Fig. 12. Voltage (x2 state variable) change over time for r=150 pu.

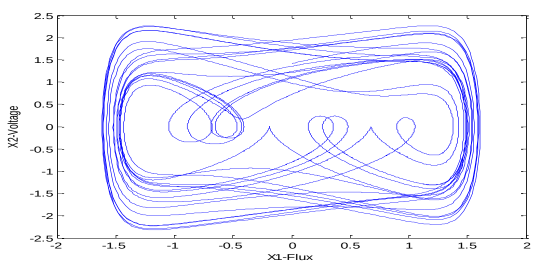

Fig. 13. Voltage–flux (x1-x2) phase portrait for r=100 pu.

In Fig. 13, all the characteristics of chaos are illustrated, including an irregular and apparently random time behaviour, as shown in Fig. 13 a phase plane trajectory that never closes on itself.

5. Sliding mode control

Sliding Mode Control is concerned with forcing one/more variable to follow a specific trajectory which is known as sliding surface [6-7]. The location of variables relative to sliding surface, which governs control law, is applied to the system. The starting point with sliding mode control is the definition of the sliding surface. For our objective, it is necessary to force the source current to be same shape in phase with source voltage. Therefore, the trajectory of line current is defined to be

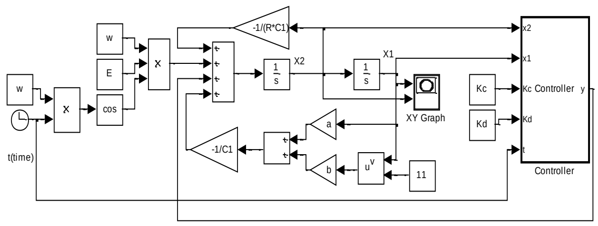

Fig. 14. Sliding mode control of chaotic ferroresonance simulink model at r=150 pu.

Control parameters are taken as Kc=1 and Kd=0.1

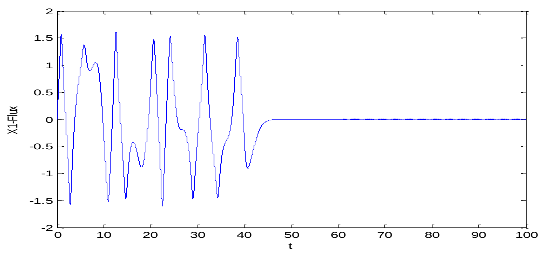

Fig. 15. Flux (x1 state variable) change over time after control.

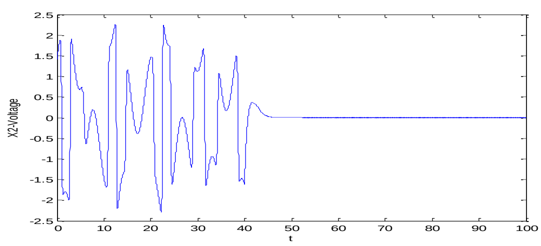

Fig. 16. Voltage (x2 state variable) change over time after control.

Controlled system equation is defined as below equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -\frac{1}{RC_1}x_2 - \frac{1}{C_1}(ax_1 + bx_1^{11}) + \omega E\cos(wt) + u \end{bmatrix}$$

where u is the control signal.

In sliding mode control of chaotic system, the sliding surface is firstly selected. An appropriate sliding surface may be chosen for needed performance as below (Eq. 3).

$$S = \mu(x_1 - x_{1ref}) + (x_2 - x_{2ref})$$

where μ is the tuning parameter. Secondly, the sliding reachability condition design is written as,

$$\dot{s} = -K_c s - K_d \operatorname{sgn}(s)$$

where K_c and K_d are positive constant design parameters.

Finally, sliding mode controller is obtained by equating both sliding surface and reachability condition. The control signal is obtained as,

$$u = -K_c s - K_d \operatorname{sgn}(s) - \mu(x_2 - \dot{x}_{1ref}) + \frac{1}{RC_1}x_2 + \frac{1}{C_1}(ax_1 + bx_1^{11}) - \omega E\cos(wt) - \dot{x}_{2ref}$$

where x_{1ref} and x_{2ref} are the reference value of the state variable of system. When the reference point of the system $x_{1ref} = 0$ and $x_{2ref} = 0$ is chosen, the control signal become like below.

$$u = -K_c s - K_d \operatorname{sgn}(s) - \mu(x_2) + \frac{1}{RC_1}x_2 + \frac{1}{C_1}(ax_1 + bx_1^{11}) - \omega E\cos(wt)$$

6. Conclusion

In this paper, the chaos control of ferroresonance phenomena in power system is investigated based on sliding mode control algorithm. The controller designed based on sliding mode control scheme is applied to the system which occurs ferroresosance phenomena in power systems. After the controller is activated at time t=40 s, the system converges to zero equilibrium point. The control of system is demonstrated in figures based on simulations. Numerical results show that the proposed method for control of chaos provides to control the chaos of ferroresonance in power systems.

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