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Calculation of the real and imaginary parts of transverse dielectric function of free electron gas with a finite electron lifetime

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The dielectric functions of a free electron gas with finite electron lifetime was calculated by K. L. Kliewer and Ronald Fuch. We present here a brief derivation for the separation of the real and imaginary parts of transverse dielectric function at finite electron lifetime for a free electron gas. These functions (real and imaginary parts) can be used for the study of optical properties of nanometals.

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1. Introduction

The real and imaginary parts of the dielectric response functions both transverse and longitudinal, for free electron-gas were first derived in detail by Lindhard [2] in the random-phase or self-consistent-field (SCF) approximation for the electron lifetime $\tau = \infty$. Then the calculations by K. L. Kliewer and R. Fuch [1] demonstrated that the finite-τ generalizations by Lindhard are incorrect and they developed the correct expressions.

But they didn't developed any expression for the real and imaginary parts of the dielectric functions. This is done in the present work for transverse dielectric function. In this calculation the quantities with a bar are dimensionless.

2. Separation of real and imaginary parts of transverse dielectric function

If one denotes the Fermi wave vector as k_F, the Fermi velocity as v_F, and by using the notation of Lindhard [2], one can write

$$z = \frac{q}{2k_f}$$

$$u' = \frac{\omega'}{qv_f} \tag{1}$$

$$\omega' = \omega + \frac{i}{\tau}$$

The expression by Kliewer and Ronald for transverse dielectric constant using the self-consistent-field technique for the finite electron lifetime τ can be written as:

$$\varepsilon_t(\vec{q}, \omega, \tau) = 1 - \frac{\omega_p^2}{\omega\omega'} f_t \tag{2}$$

where:

$\omega_p = \left(\frac{4\pi e^2 n_0}{m}\right)^{\frac{1}{2}}$ is the classical resonance frequency (the plasma frequency) of the gas and n_0 is the equilibrium particle density .

Furthermore:

$$f_t = \frac{3}{8}(z^2 + 3u'^2 + 1) - \frac{3}{32z}\left[\left[1 - (z - u')^2\right]^2 \ln\left(\frac{z - u' + 1}{z - u' - 1}\right) + \left[1 - (z + u')^2\right]^2 \ln\left(\frac{z + u' + 1}{z + u' - 1}\right)\right] \tag{3}$$

In the following we shall use the dimensionless quantities with a bar:

$$z = \frac{q}{2k_f} = \frac{\overline{q}}{2}$$

$$\omega' = \frac{E_f}{\hbar}\overline{\omega} + \frac{i}{\tau} = \frac{E_f}{\hbar}\overline{\omega} + i\gamma \tag{4}$$

$$u' = \frac{\omega'}{qv_f} = \frac{\overline{\omega} + i\gamma\frac{\hbar}{E_f}}{\frac{\hbar k_f v_f}{E_f}\overline{q}} = \frac{\overline{\omega} + ia}{b\overline{q}}$$

where:

$$\frac{1}{\tau}=\gamma \quad , \quad \overline{q}=\frac{q}{k_f} \quad , \quad \overline{\omega}=\frac{\hbar\omega}{E_f} \quad , \quad a=\frac{\gamma\hbar}{E_f} \quad ,$$

$$b=\frac{\hbar k_f v_f}{E_f} \tag{5}$$

so we can rewrite the equation (3) as:

$$f_t=\frac{3}{8}\left(\left(\frac{\overline{q}}{2}\right)^2+3\left(\frac{\overline{\omega}+ia}{b\overline{q}}\right)^2+1\right)-\frac{3}{16\overline{q}}\left[\begin{array}{l}\left[1-\left(\frac{\overline{q}}{2}-\frac{\overline{\omega}+ia}{b\overline{q}}\right)^2\right]^2\ln\left(\dfrac{\frac{\overline{q}}{2}-\frac{\overline{\omega}+ia}{b\overline{q}}+1}{\frac{\overline{q}}{2}-\frac{\overline{\omega}+ia}{b\overline{q}}-1}\right)+\\ \left[1-\left(\frac{\overline{q}}{2}+\frac{\overline{\omega}+ia}{b\overline{q}}\right)^2\right]^2\ln\left(\dfrac{\frac{\overline{q}}{2}+\frac{\overline{\omega}+ia}{b\overline{q}}+1}{\frac{\overline{q}}{2}+\frac{\overline{\omega}+ia}{b\overline{q}}-1}\right)\end{array}\right] \tag{6}$$

It is convenient to use a new notation:

$$f_t=\frac{3}{8}(A+iB)-\frac{3}{16\overline{q}}\left[(F+iG)\ln\frac{x_1-iD}{x_1'-iD}+(H+iI)\ln\frac{x_2+iD}{x_2'+iD}\right] \tag{7}$$

where:

$$A=\frac{\overline{q}^2}{4}+\frac{3(\overline{\omega}^2-\overline{a}^2)}{b^2\overline{q}^2}+1$$

$$B=\frac{6\overline{\omega}a}{b^2\overline{q}^2}$$

$$x_1=\frac{\overline{q}}{2}-\frac{\overline{\omega}}{b\overline{q}}+1$$

$$x_1'=\frac{\overline{q}}{2}-\frac{\overline{\omega}}{b\overline{q}}-1$$

$$x_2=\frac{\overline{q}}{2}+\frac{\overline{\omega}}{b\overline{q}}+1$$

$$x_2'=\frac{\overline{q}}{2}+\frac{\overline{\omega}}{b\overline{q}}-1$$

$$F=1+C^4+D^4-6C^2D^2-2C^2+2D^2$$

$$G=4(CD^3-DC^3+CD)$$

$$H=1+E^4+D^4-6E^2D^2-2E^2+2D^2$$

$$I=4(ED^3+DE^3-ED)$$

with:

$$C=\frac{\overline{q}}{2}-\frac{\overline{\omega}}{b\overline{q}}$$

$$D=\frac{a}{b\overline{q}}$$

$$E=\frac{\overline{q}}{2}+\frac{\overline{\omega}}{b\overline{q}}$$

We can rewrite Eq. (7) as:

$$f_t=\frac{3}{8}(A+iB)-\frac{3}{16\overline{q}}\left[(F+iG)\left(\ln(x_1-iD)-\ln(x_1'-iD)\right)+(H+iI)\left(\ln(x_2+iD)-\ln(x_2'+iD)\right)\right]$$

Now by using of the following relation:

$$\ln(x\pm iy)=\frac{1}{2}\ln\left(x^2+y^2\right)\pm i\,arctg\,\frac{y}{x}$$

in the Eq. (7) one obtains:

$$f_t=\frac{3}{8}(A+iB)-\frac{3}{16\overline{q}}\left[\begin{array}{l}(F+iG)\left(\frac{1}{2}\ln\left(x_1^2+D^2\right)-i\,arctg\,\frac{D}{x_1}-\frac{1}{2}\ln\left(x_1'^2+D^2\right)+i\,arctg\,\frac{D}{x_1'}\right)+\\ (H+iI)\left(\frac{1}{2}\ln\left(x_2^2+D^2\right)+i\,arctg\,\frac{D}{x_2}-\frac{1}{2}\ln\left(x_2'^2+D^2\right)-i\,arctg\,\frac{D}{x_2'}\right)\end{array}\right] \tag{8}$$

Again we use of a simple notation for rewriting Eq. (8):

$$J=\frac{1}{2}\ln(x_1^2+D^2)$$

$$K=arctg\left(\frac{D}{x_1}\right)$$

$$J'=\frac{1}{2}\ln(x_1'^2+D^2)$$

$$K'=arctg\left(\frac{D}{x_1'}\right)$$

$$L=\frac{1}{2}\ln(x_2^2+D^2)$$

$$M=arctg\left(\frac{D}{x_2}\right)$$

$$L'=-\frac{1}{2}\ln(x_2'^2+D^2)$$

$$M'=arctg\left(\frac{D}{x_2'}\right)$$

We can rewrite Eq. (8) as:

$$f_t=\frac{3}{8}(A+iB)-\frac{3}{16\overline{q}}\left[(F+iG)\left(J-iK-J'+iK'\right)+(H+iI)\left(L+iM-L'-iM'\right)\right] \tag{9}$$

Now by separating the real and imaginary parts of Eq. (9) one gets:

$$f_t = f_t' + i f_t'' \qquad (10)$$

where:

$$f_t' = \frac{3}{8}A - \frac{3}{16\overline{q}}(FJ - FJ' + GK - GK' + HL - HL' - IM + IM')$$

(11)

$$f_t'' = \frac{3}{8}B - \frac{3}{16\overline{q}}(-FK + FK' + GJ - GJ' + MH - M'H + LI - IL')$$

Eq. (1) can be rewritten as:

$$\varepsilon_T(\overline{q}, \overline{\omega}, \tau) = 1 - \frac{s}{\overline{\omega}(\overline{\omega} + it)} f_t \qquad (12)$$

where:

$$s = \frac{\hbar^2 \omega_p^2}{E_f^2}$$

$$t = \frac{\hbar\gamma}{E_f}$$

Finally, we get the transverse dielectric function as:

$$\varepsilon_T(\overline{q}, \overline{\omega}, \tau) = 1 - \frac{s}{\overline{\omega}^2 + t^2} f_t' + \frac{st f_t''}{\overline{\omega}(\overline{\omega}^2 + t^2)} - i\left(\frac{s f_t''}{\overline{\omega}^2 + t^2} + \frac{st f_t'}{\overline{\omega}(\overline{\omega}^2 + t^2)}\right) \qquad (12)$$

By separating the real and imaginary parts of Eq. (12) one obtains:

$$\begin{cases} \mathrm{Re}(\varepsilon_T(\overline{q}, \overline{\omega}, \tau)) = \varepsilon_T'(\overline{q}, \overline{\omega}, \tau) = 1 - \dfrac{s}{\overline{\omega}^2 + t^2} f_t' + \dfrac{st}{\overline{\omega}(\overline{\omega}^2 + t^2)} f_t'' \\ \mathrm{Im}(\varepsilon_T(\overline{q}, \overline{\omega}, \tau)) = \varepsilon_T''(\overline{q}, \overline{\omega}, \tau) = -\left(\dfrac{s f_t''}{\overline{\omega}^2 + t^2} + \dfrac{st f_t'}{\overline{\omega}(\overline{\omega}^2 + t^2)}\right) \end{cases} \qquad (13)$$

3. Conclusion

In this paper we have calculated the real and imaginary parts of the transverse dielectric function for a free electron gas at finite electron lifetime. These results can be used for the study of optical properties of nano-metals.

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