Bright-Dark combo optical solitons with non-local nonlinearity in parabolic law medium

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In this paper a new type of combined optical solitons are derived in the weakly nonlocal nonlinear parabolic law medium for the first time. The nonlocal nonlinear Schrödinger equation is investigated analytically. The combined bright-dark solitons and the corresponding existence conditions are reported. The presented results have important applications in nonlinear optics and Bose-Einstein condensates.

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1. Introduction

The nonlocal nonlinear response of the medium at a given point determines on optical pulse intensity at that point and its vicinity [1-4]. The nonlocality of nonlinearity, including three types that are weakly nonlocality, general nonlocality and strongly (highly) nonlocality [1, 4, 5], arises in various nonlinear systems, such as the Bose-Einstein condensates (BEC), nonlinear optics and plasmas [4,6].

Optical solitons are widely applied in the area of telecommunications and electromagnetics [7]. Current studies are mainly focus on the optical bright or (and) dark solitons in the nonlinear media [8-20], while there are very few researches to the combined solitons [21,22], which include the bright-dark solitons, singular-bright solitons, singular-dark solitons and singular-bright-dark solitons. So the key idea of this work is to construct analytical combined solitons in a medium with weakly nonlocal nonlinearity and parabolic law nonlinearity. The combined optical bright-dark solitons will be constructed in this paper.

2. Governing equation

In the presence of weakly nonlocal nonlinearity, the governing equation for the propagation of combined solitons through parabolic law medium is given by the following nonlocal nonlinear Schrödinger equation (NNLSE) [23-25]

$$iu_t + au_{xx} + b|u|^2 u + c|u|^4 u + d(|u|^2)_{xx} u = 0 \tag{1}$$

where the dependent variable $u(x,t)$ represent the normalized slowly varying amplitude of the electric field, while the independent variables x and t represent the distance and time respectively. Here the constants a and d are the parameters of group velocity dispersion (GVD) and weakly nonlocal nonlinearity [23, 24], while the terms with b and c are due to the parabolic law (cubic-quintic) nonlinearity.

In the previous work [25], we obtained the singular, dark and bright soliton solutions to Eq. (1) by using the Jacobi's elliptic function method, Riccati's equation approach and ansatz scheme. In this study, we will report a new type of combined optical bright-dark solitons. Additionally, the corresponding existence condition will be given.

3. Exact combined soliton solutions to Eq. (1)

In order to get explicit solutions to Eq. (1), we first make the hypothesis in the form

$$u(x,t) = P[\eta(x,t)]\exp[i\phi(x,t)] \tag{2}$$

where $\eta = B(x - vt)$ and $\phi(x,t) = -\kappa x + \omega t + \theta$. $P(\eta)$ and $\phi(x,t)$ are the amplitude and phase components of the soliton to be determined later. Here B, v, κ, ω, and θ

are the real constants that represent the width, velocity, frequency, wave number and phase constant of the soliton.

Substituting Eq. (2) into Eq. (1) yields

$$i(-BvP'+i\omega P)+a(B^2P''-2i\kappa BP'-\kappa^2P)+bP^3 + cP^5+2dB^2(PP'^2+P^2P'')=0 \tag{3}$$

Separating the real and imaginary parts yields

$$v=-2a\kappa \tag{4}$$

$$(2dP^2+a)B^2P''+2dB^2PP'^2 - -(\omega+a\kappa^2)P+bP^3+cP^5=0 \tag{5}$$

Eq. (4) gives the velocity of the soliton, while Eq. (5) will be integrated to get the combined bright-dark soliton solutions by employing the ansatz method.

First we introduce the hypothesis in the form

$$P(\eta)=M+L\operatorname{sech}\eta+iN\tanh\eta \tag{6}$$

where M, L, and N are real constants that represent the amplitude of bright soliton, the amplitude of dark soliton and the grayness of the soliton.

From Eq. (6), we have

$$P^2=M^2-N^2+2ML\operatorname{sech}\eta+2iMN\tanh\eta+ +(L^2+N^2)\operatorname{sech}^2\eta+2iNL\operatorname{sech}\eta\tanh\eta \tag{7}$$

$$\begin{aligned}P^3&=M^3-3MN^2+3L(M^2-N^2)\operatorname{sech}\eta+\\&+iN(3M^2-N^2)\tanh\eta+3M(L^2+N^2)\operatorname{sech}^2\eta+\\&+L(3N^2+L^2)\operatorname{sech}^3\eta+6iMNL\operatorname{sech}\eta\tanh\eta+\\&+iN(3L^2+N^2)\operatorname{sech}^2\eta\tanh\eta\end{aligned} \tag{8}$$

$$\begin{aligned}P^5&=M^5+5MN^4-10M^3N^2+\\&+5L(M^4+N^4-6M^2N^2)\operatorname{sech}\eta\\&+iN(5M^4-10M^2N^2+N^4)\tanh\eta+\\&+10M(M^2L^2-N^4+M^2N^2-3N^2L^2)\operatorname{sech}^2\eta+\\&+10L(M^2L^2-N^4+3M^2N^2-L^2N^2)\operatorname{sech}^3\eta+\\&+5M(L^4+N^4+6N^2L^2)\operatorname{sech}^4\eta+\\&+L(L^4+5N^4+10L^2N^2)\operatorname{sech}^5\eta+\\&+20iMLN(M^2-N^2)\operatorname{sech}\eta\tanh\eta+\\&+2iN(15M^2L^2+5M^2N^2-5L^2N^2-N^4)\operatorname{sech}^2\eta\tanh\eta+\\&+20iMNL(L^2+N^2)\operatorname{sech}^3\eta\tanh\eta+\\&+iN(5L^4+10L^2N^2+N^4)\operatorname{sech}^4\eta\tanh\eta\end{aligned} \tag{9}$$

$$P'=iN\operatorname{sech}^2\eta-L\operatorname{sech}\eta\tanh\eta \tag{10}$$

$$P''=L\operatorname{sech}\eta-2iN\operatorname{sech}^2\eta\tanh\eta-2L\operatorname{sech}^3\eta \tag{11}$$

Substituting Eqs. (6) - (11) into Eq. (5), using the homogeneous balance principle, i.e. letting the coefficients of each term of $\operatorname{sech}^n\eta\tanh^m\eta$ ($n=0,1,2,3,4,5$, $m=0,1$) to zero gives

$$M(-\omega-a\kappa^2+bM^2-3bN^2+cM^4+ +5cN^4-10cM^2N^2)=0 \tag{12}$$

$$L[-\omega-a\kappa^2+aB^2+3b(M^2-N^2)+ +5c(M^4+N^4-6M^2N^2)+2dB^2(M^2-N^2)]=0 \tag{13}$$

$$iN[-\omega-a\kappa^2+b(3M^2-N^2)+ +c(5M^4-10M^2N^2+N^4)]=0 \tag{14}$$

$$iN\begin{bmatrix}-2aB^2+b(3L^2+N^2)+2c(15M^2L^2+\\+5M^2N^2-5L^2N^2-N^4)\\+4dB^2(L^2-M^2+N^2)+2dB^2L^2\end{bmatrix}=0 \tag{15}$$

$$L\begin{bmatrix}-2aB^2+b(3N^2+L^2)+\\+10c(M^2L^2-N^4+3M^2N^2-L^2N^2)+\\+2dB^2(7N^2+L^2-2M^2)+2dB^2(2N^2+L^2)\end{bmatrix}=0 \tag{16}$$

$$M\begin{bmatrix}3b(L^2+N^2)+\\10c(M^2L^2-N^4+M^2N^2-3N^2L^2)+\\+4dB^2(L^2+2N^2)+2dB^2L^2\end{bmatrix}=0 \tag{17}$$

$$2iMNL[3b+10c(M^2-N^2)+2dB^2]=0 \tag{18}$$

$$5M[c(L^4+N^4+6N^2L^2)-2dB^2(N^2+L^2)]=0 \tag{19}$$

$$L[c(L^4+5N^4+10L^2N^2)-6dB^2(3N^2+L^2)]=0 \tag{20}$$

$$20iMNL[c(L^2+N^2)-dB^2]=0 \tag{21}$$

$$iN[c(5L^4+10L^2N^2+N^4)- -6dB^2(N^2+3L^2)]=0 \tag{22}$$

To begin with, from Eq. (20) and (22), we have

$$N=\pm L \tag{23}$$

then Eqs. (19)-(21) give the relations

$$M(2cN^2-dB^2)=0 \tag{24}$$

$$L(2cN^2-3dB^2)=0 \tag{25}$$

$$iMNL(2cN^2-dB^2)=0 \tag{26}$$

$$iN(2cN^2 - 3dB^2) = 0 \tag{27}$$

From Eqs. (24)-(27), we get

$$M = 0 \tag{28}$$

$$B = \pm\sqrt{\frac{2c}{3d}}N \tag{29}$$

Additionally, solving Eqs. (12) - (18), we have

$$\omega = aB^2 - 3N^2b + 5cN^4 - 2N^2dB^2 - a\kappa^2 \tag{30}$$

$$\omega = -N^2b + cN^4 - a\kappa^2 \tag{31}$$

$$-aB^2 + 2N^2b - 6cN^4 + 5N^2dB^2 = 0 \tag{32}$$

It needs to be noted that upon balancing the two values of the wave number from Eqs. (30) and (31) also yields the same relation as given by (32).

Hence, the combined bright-dark soliton solutions for the parabolic law medium with weakly nonlocal nonlinearity are given by

$$u(x,t) = \begin{Bmatrix} L\operatorname{sech}[B(x-vt)] + \\ + iN\tanh[B(x-vt)] \end{Bmatrix} \exp\{i[-\kappa x + \omega t + \theta]\} \tag{33}$$

where the soliton velocity and soliton width are given by Eqs. (4) and (29) respectively, while the wave number is given by Eq. (30) or Eq. (31) and additionally the relation for bright soliton amplitude and dark soliton amplitude is given by Eq. (23). Finally, the corresponding existence condition is given by Eq. (32).

4. Conclusions

This paper addressed the nonlinear dynamics for optical solitons in medium with the weakly nonlocal nonlinearity and parabolic law nonlinearity that is modeled by the NNLSE, i.e. Eq. (1). A new type of combined optical bright-dark solitons is obtained. The integration tool is ansatz scheme.

Ansatz method is a very powerful method to construct analytical solutions to the nonlinear evolution equations and is worth studying further. In future studies, Other two laws of nonlinearity that are polynomial law nonlinearity and dual-power law nonlinearity will be considered. The new combined optical solitons that are singular-bright solitons, singular-dark solitons and singular-bright-dark solitons will be studied.

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