# Ballistic target tracking rotary reflection mirror calculation model and optical performance analysis

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To improve target tracking performance and target information extraction method in ballistic synchronous tracking system, this paper proposes a combination method that using single high-speed cameras and rotary reflection mirror to realize target synchronous tracking, sets up the calculation model of ballistic target tracking rotary reflection mirror; researches target tracking geometry calculation algorithm according to the relation of rotary reflection mirror, high-speed cameras and target flying position; deduces the calculation relation function of rotary reflection mirror rotation angle and angular velocity; analyzes the influence factors of target tracking system and target optical properties in detection field of view; and gives the critical parameters calculation method and eliminate strong background luminance effect measurement. Through theoretical calculation and the synchronous tracking experiment, the changing curves and function relation of rotary reflection mirror rotation angle and angular velocity and the time were given under different target flying velocity, and using the testing data of the synchronous target tracking experiment to verify the calculation model of ballistic target tracking rotary reflection mirror.

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## 1. Introduction

In the exterior ballistic target flight alttitude test field, the dynamic information of flying target is usually focused in whole exterior ballistic trajectory so that we can realize target locating and tracking. At present, in order to track flight target, we use multi-high-speed cameras synchronous tracking layout in exterior ballistic along the target flight direction, and form multi position test, when the target flying past the high-speed camera's field of view, high-speed cameras can record the images of projectiles target in turn, and then, by analyzing each camera's images and using the splicing method to dispose target image, the whole moving images of projectile target in ballistic will be obtained. However, due to restrictions by the shutter speed of field of view, it is difficult to reach the full trajectory tracking [1]. Meanwhile, multi-high-speed cameras images analysis requires sophisticated image processing technology, the cost of multiple high-speed cameras is very high, so, the method of multi-high-speed cameras cannot meet current demand [2].

References [3-5] presents another method to set up the camera on the tracking platform to real-time track target, and sue single high-speed cameras gather the flying target continuously, this method is easy to track lost. References [6-7] refer to the use of a combination of single high-speed cameras with wide-angle lenses, the high-speed camera is placed away from the schedule ballistic trajectory to capture flying target, but, the images quality of this method is very poor that cannot achieve the target information extraction, the target information also lose. So, it is very necessary to research a new method to solve the reliability and stability of ballistic target tracking and reduce costs, this paper proposes a combination method by using single high-speed cameras and rotary reflection mirror, and set up its calculation model and optical performance.

## 2. Principle of synchronous target tracking in schedule ballistic

Ballistic synchronous tracking system is to achieve dynamic target tracking and monitoring in whole ballistic trajectories, and realize the target tracking in different area. So that we can master and understand dynamic target's velocity, flying angle, flying attitude, and so on. Dynamic target has randomness and particularity when it flying in ballistic orbit, if we use a single camera tracking method, the target information may be lost. To observe and track high-speed flying target, we must set up the synchronous tracking control system and tracking model, Fig. 1 is new design the synchronous target tracking system in schedule ballistic. Point A is the layout position of cannon; Point B is the start position when target flying into the tracking detection view, point C is the end position.

To capture and track the dynamic target accurately, we use cannon's flame information to design the synchronous trigger device to provide synchronous track command for target tracking platform. Synchronous trigger device was layout in cannon's position. The tracking platform is consisted of high speed camera, reflection spinning mirror and image acquisition. In Fig. 1, $L$ is the distance between point D and tracking platform, $(-\theta, \theta)$ is the view field of rotary reflection mirror. When the length of track orbit BC and the view field of rotary reflection mirror were ensure,

$$L = (BC \cdot ctg\theta)/2.$$

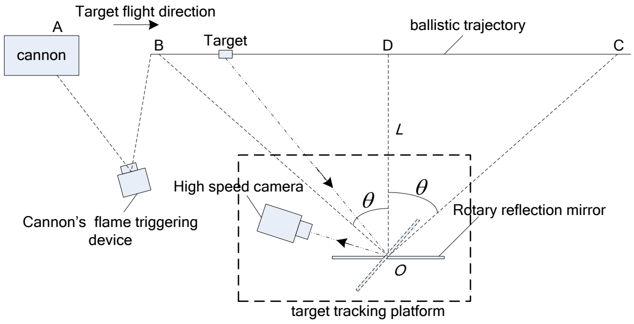



*Fig. 1. Schematic diagram of ballistic synchronous tracking system in schedule ballistic*

From Fig. 1, if we want to realize the target real-time tracking in BC range, the rotation range of rotary reflection mirror range should be from $-\theta$ to $\theta$. We use computer to storage and processing target image and gain the target information, and then, use this information to control reflection spinning mirror and realize dynamic target tracking.

## 3. Target tracking geometry calculation algorithm based on rotary reflection mirror

Fig. 2 is schematic diagram of trajectory synchronous tracking based on rotary reflection mirror. Assuming the air resistance is negligible. The primary optical axis of high-speed cameras optical system is parallel to the schedule ballistic trajectory.

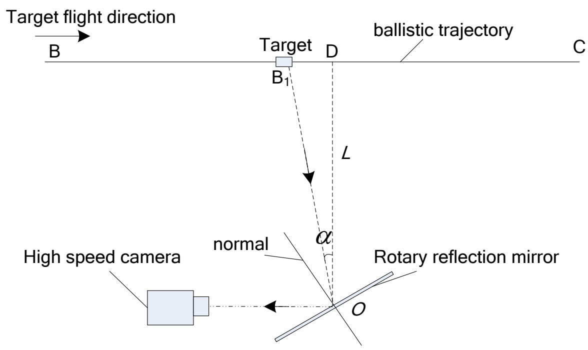



*Fig. 2. The schematic diagram of trajectory synchronous tracking based on rotary reflection mirror*

Based on Fig. 1 and Fig. 2, Suppose, $v$ is the target flying velocity, $\delta$ is the reflection mirror rotation angle, when $vt < B_1D$, $\delta$ can be calculated by formula (1).

$$\delta = \frac{1}{2}\left[\alpha + arctg(vt - L\tan\alpha)/L\right] \quad (1)$$

When $vt > B_1D$, $\delta$ can be calculated by formula (2).

$$\delta = \frac{1}{2}\left[\alpha - arctg(L\tan\alpha - vt)/L\right] \quad (2)$$

If $\omega$ is the angular velocity of rotary reflection mirror, $t$ is target flying time, we can calculate the relation between $\omega$ and $t$, when $vt < B_1D$, $\omega$ can be calculated by formula (3).

$$\omega = \frac{v}{2L}\cos^2\left[arctg(vt - L\tan\alpha)/L\right] \quad (3)$$

When $vt > B_1D$, $\omega$ can be calculated by formula (4).

$$\omega = \frac{v}{2L}\cos^2\left[arctg(L\tan\alpha - vt)/L\right] \quad (4)$$

We use formula (1), (2), (3) and (4) to calculate the reflection mirror rotation angle and the angular velocity of rotary reflection mirror at any time.

## 4. Optical performance of ballistic synchronous tracking system

In ballistic synchronous tracking system, whether can obtain target image information that come from rotary reflection mirror to high-speed camera, it depends on the sky background illumination in effective detection view filed and the contrast, and the main image component is area array CCD in high-speed camera, so, the image performance of area array CCD is main influence factor, to improve the optical performance of ballistic synchronous tracking system, we must research its optical characteristic.

Based on photometry theory [8-9], we know the scattered light of flying target is mainly determined by sunlight and ground reflection. If the background luminance on the area array CCD photosensitive surface is $E_b$, it can expressed by formula (5).

$$E_b = B_b \frac{\pi}{4} \tau \left(\frac{D}{f}\right)^2 kl \quad (5)$$

In (5), $B_b$ is the sky background luminance, $f$ is focus of high-speed camera lens, $\tau$ is transmittance rate, $D$ is aperture size of lens, $k$ and $l$ respectively are length and width of area array CCD photosensitive surface,

$E$ is the brightness target oneself in area array CCD photosensitive surface.

$$E = \frac{\rho}{\pi} E_0 A_1 \frac{\pi}{4} (\frac{D}{f})^2 \tau \quad (6)$$

In (6), $\rho$ is reflectance of target to the sunlight, $A_1$ is effective reflection area of target, $E_0$ is the illuminance of target that come from sunlight. According to the definition of contrast [10], if $C_M$ is the contrast of ballistic synchronous tracking system, and then, $C_M$ can gain by formula (7).

$$C_M = \frac{E - E_b}{E_b} = \frac{\frac{\rho}{\pi} E_0 A_1 - B_b \cdot k \cdot l}{B_b \cdot k \cdot l} \quad (7)$$

In (7), when $C_M > 0$, the target can be captured and recognized from image.

According to the process of targets flying the image view of high-speed camera, we can divide into three states. First, the target is just entering the image view of lens; second, the target is full entering the image view of lens; last, the target is leaving the image view of lens. To three states, area array CCD photosensitive surface will obtain different radiation energy from target oneself.

When target start into the image view of lens, the area array CCD photosensitive surface gain the information is very small, it can calculate by formula (8).

$$I_1 = \Delta K \cdot (l - \frac{L}{f} \Delta K) \cdot \kappa \cdot (E - E_b) / v \quad (8)$$

In (8), $\kappa$ is average spectral responsivity of area array CCD, $\Delta K$ is the image edge length in area array CCD photosensitive surface [11].

When target full enter the image view of lens, the area array CCD photosensitive surface gain the information is maximum, its calculation expression is:

$$I_2 = \Delta A \cdot \kappa \cdot (E - E_b) \cdot f \cdot d / vL \quad (9)$$

$\Delta A$ is photosensitive area when it is in the CCD light-sensitive surface.

When target leaves the image view of lens, we use $I_3$ to express its function, because target just enter the image view of lens and leave is the same, and then, $I_3 = I_1$, through analyzing, we can due derive the output current function.

$$I_o = [2\Delta K(l - \frac{L}{f} \Delta K) + \Delta A f d L^{-1}] \kappa (E - E_b) / v \quad (10)$$

According to the influencing factors of tracking system and target characteristics analysis, to stable tracking target, we must calculate the key parameters of ballistic synchronous tracking system.

## 5. Key parameters calculation of ballistic synchronous tracking system

### 5.1 Maximum discrete interval of flight target

Assume, $v$ is the target flight velocity, $l$ is the length of target, $d$ is the diameter of target, $a \times b$ is the area of high-speed camera's field of view, $b$ is detection visual field, Fig. 3 shows difference states when target flying into the view of area array CCD.

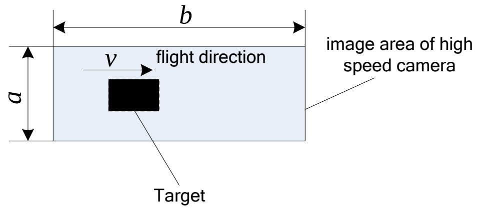
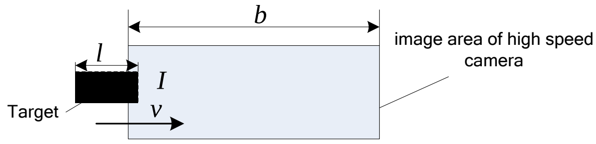
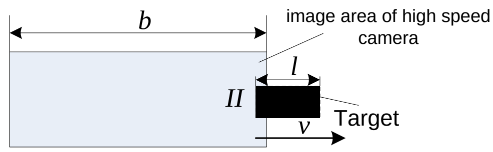



*Fig. 3. Sketch map of target moving in the field of view of high-speed camera*

If position $I$ is starting point when target just enter the image field of view, position $II$ is leaving point, as shown in Fig. 4. According to the tracking system geometry model, if $t$ is the time that target can image in area array CCD photosensitive surface, but target cannot leave area array CCD photosensitive surface, the maximum discrete intervals velocity is $\Delta v$, and then, $\Delta v = (0.3b - l)/t$, it make high speed camera gain more than three pixel target imaging information, it ensure target can stable track in ballistic synchronous tracking system.




(a) when target just enter the image field of view




(b) when target leave the image field of view

*Fig. 4. Sketch map of target moving in two edge field of view of high-speed camera*

When the length of the target is more than 0.3 times the length of detection visual field, the target tracking system is unstable, so, to stable tracking target, we must adjust the distance between point D and tracking platform based on Fig. 1.

### 5.2 Scanning rate of area array CCD

Scan rate was defined by using the ratio that target flight velocity and vertical distance $L$, suppose, $\gamma$ is scan rate, it can expressed by formula (11).

$$\gamma = v / L \tag{11}$$

To stable tracking target in scheduled tracking trajectory, we must ensure scan rate. Taking target's own gravitational influence into account and the acceleration of gravity when target flying. If $t_0$ is the flight time, $\Delta s$ is the vertical moving distance of the target under the influence of gravitational acceleration, and then, $\Delta s = g t_0^{\,2}/2$ [12].

To ensure that the target tracking will not be lost, the field of view of high and low direction of rotary reflection mirror should be more than $\Delta s$. So, the minimum scan rate can be calculated by formula (13).

$$\gamma_{\min} = BC \cdot \left(\frac{g}{aL^2}\right)^{1/2} \tag{12}$$

## 6. Calculation and experimental analysis

### 6.1 The relationship between the rotation angle of rotary reflection mirror and flight time

According to the calculation model of ballistic synchronous tracking system, if the target flight velocity is 1000m/s, the distance of start position and final position is 1000m, the vertical distance from the tracking platform to ballistic is 340m. Based on the formula (1), (2), (3) and (4), we can calculate the changing curve of rotation angle of rotary reflection mirror ($\theta$) and target flight time($t$), as show Fig. 5.

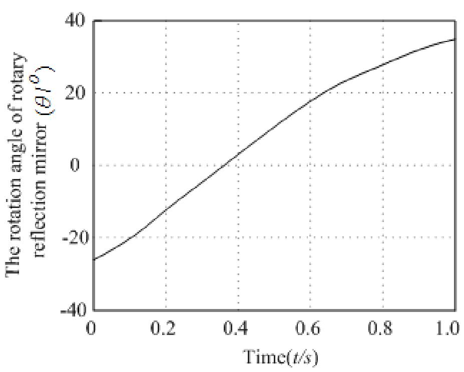



*Fig. 5. The changing curve of reflection rotation angle and flight time*

From Fig. 5, we know when the longer the target flight time is, the larger of rotation angle.

### 6.2 The relation between the angular velocity of rotary reflection mirror and flight time

Fig. 6 is the changing curve of angular velocity of rotary reflection mirror and flight time under difference flight velocity.

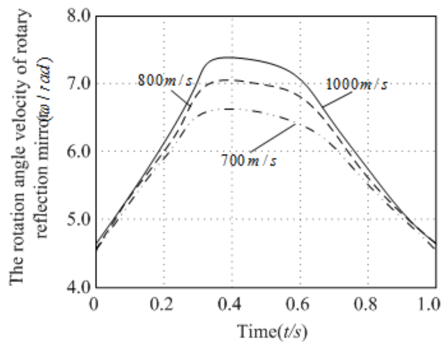



*Fig. 6. The changing curve of angular velocity of rotary reflection mirror and flight time under difference flight velocity*

According to Fig. 5 and Fig. 6, along with the increase of flight time, the rotation angle of rotary reflection mirror increases. Angular velocity of rotary reflection mirror is small when the target starts entering the field of view. When the target locates the shortest distance to tracking platform, angular velocity of rotary reflection mirror reaches the maximum and then decreases. It show that if we want to ensure stable tracking target, the servo motor that control rotary reflection mirror need to adjust the rotational velocity of rotary reflection mirror at different position.

According to the view size of rotary reflection mirror and the trajectory of target flight, the field of view of rotary reflection mirror is small in the middle of ballistic because the distance between the target and the tracking system is relatively short, and the flight time is shorter at a certain flight speed, to track target, we must improve rotation angle of rotary reflection mirror. In addition, when the target flight velocity is reduced, the angular speed of rotary reflection mirror has also become smaller.

### 6.3 Relationship between output signal of high-speed camera and background illumination

According to contrast calculation function, if the contrast is low, the tracking systems will not be able to extract target information from the background signal in area array CCD output system, and the tracking performance will weaken, which is mainly reflected in background illumination.

Based on formula (5) to (12), Fig. 7 gives the output signal of tracking system in different sizes and different background illumination.

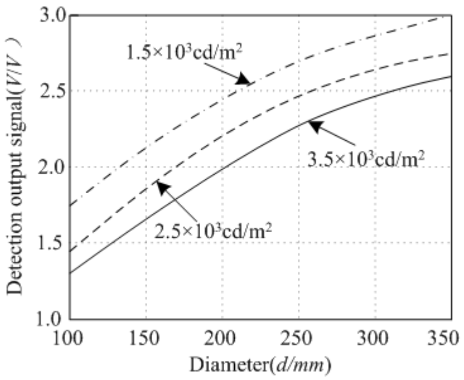



*Fig. 7. The output signal of ballistic synchronous tracking system in different sizes and different background illumination*

Fig. 7 shows that the output signals are relatively weak when background illumination is relative high or low, it shows that area array CCD sensor of high-speed camera's maybe be saturated or cannot gather target information, which make the contrast is reduced. In addition, if we taking into account the target size, the bigger of target size, the higher of contrast.

### 6.4 Spectral filtering technology to improve strong background illumination influence

Ballistic synchronous tracking system main use sky as background light to work, so, the background illumination will affect target tracking performance. From formula (5) to (10), to improve strong background illumination influence, we use spectral filtering technology.

In ballistic synchronous tracking system, the target mainly relies on the sunlight reflection because the space target and background luminance is a process of the spatial and temporal variation. It can be known from the calculation model of target luminance and sky background luminance that the detection performance of synchronous tracking system is also related to the spectral filtering parameter of optical lens when the detection system design parameters are known. The detection ability of synchronous tracking system is affected by background signal's filter coefficient and the target signal's filter coefficient in different filtering methods. The normalized photon flux curve of sky background light is given on the basis of the sunlight. So, we choose the cut-off wavelength is 0.65um to filter the strong background luminance.

But for different test ranges, the contribution to the detection ability of ballistic synchronous tracking system is different even though adopting the same filter method when the background spectral characteristics are different. Therefore, in the design of image optical system, we select the appropriate spectrum filtering methods to improve the detection ability of ballistic synchronous tracking system in the strong background luminance.

### 6.5 Experiment and analysis

In order to verify the stability of ballistic synchronous tracking system, we choose to use infrared high-speed cameras to gain dynamic target image information. According to the rotary reflection mirror principle and synchronous control method, we design synchronous control system and image target tracking platform，optical lens is 70-300mm, average spectral responsivity of area array CCD is 0.81, the image area of area array CCD photosensitive surface is 10mm×13.8mm, the size of rotary reflection mirror is 45mm × 36mm, tracking distance about is 1800m, the distance between schedule ballistic and tracking platform about is 400m, theoretical velocity of target flying velocity about is 450m/s. To verify the calculation model and method of ballistic target tracking rotary reflection mirror, we compare actual control angular velocity and theoretic control angular velocity, and calculate their error, if the error meet key parameters that show the calculation model is correct. Table 1 is testing data and theoretic control data.

*Table 1. Actual control angular velocity and theoretic control angular velocity*

| Track control moment (*t/s*) | Actual control angular velocity ($\omega / rad$) | Theoretic control angular velocity ($\omega / rad$) |
|---|---|---|
| 0.2 | 3.55 | 3.5 |
| 0.8 | 4.71 | 4.6 |
| 1.0 | 5.38 | 5.4 |
| 1.5 | 6.82 | 7.0 |
| 1.8 | 6.64 | 6.5 |
| 2.3 | 4.76 | 4.5 |
| 2.9 | 3.49 | 3.3 |

From Table 1, the state of angular velocity of rotary reflection mirror is changing in the process of target tracking. For the same tracking velocity of target, angular velocity of rotary reflection mirror at the beginning and end of trajectory are smaller than the one in the middle. According to the error data, we calculate the discrete interval of flight target, the discrete intervals velocity $\Delta v$ is meet control demand, and the scan rate $\gamma$ about is 3.28, it make the tracking system can stable tracking target. The experimental results are in agreement with the theory calculation results.

## 7. Conclusions

According to the need of ballistic target tracking, this paper researches the method of using single high-speed cameras and rotary reflection mirror to set up tracking model, analyzes target optical properties, and give the calculation method of key parameters of the tracking systems. By theoretical calculations and experiment, as a result, angular velocity of rotary reflection mirror is a parabola throughout the tracking trajectory in different target flight velocity, further, when tracking platform is nearest from the ballistic, angular speed is the fastest while the lowest at the initial position and the end position with an increasing trend. In addition, the test environment background illumination of ballistic synchronous tracking system is closely related to tracking performance, especially at strong background, which results in contrast decline. The theoretical calculation method of this paper provides the basis for the design of the remote dynamic target tracking.

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