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Analytical study of Thirring solitons in spatially inhomogeneous birefringent fibers

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This paper studies Thirring optical solitons in spatially inhomogeneous birefringent fibers with Kerr law nonlinearity and spatio-temporal dispersion. By virtue of the ansatz approach, analytical Thirring optical 1-solitons are constructed, which include bright, dark and singular solitons. The coefficient constraint conditions that need to hold for the existence of these Thirring optical solitons are reported.

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1. Introduction

Thirring optical solitons are the outcome of delicate equilibrium between between cross-phase modulation (XPM) and group velocity dispersion (GVD) [1-3]. Recently, the study of Thirring optical solitons in homogeneous birefringent fibers has attracted much attention [1-6], the dynamics of Thirring optical solitons propagating in birefringent fibers is modeled by the vector coupled nonlinear Schrödinger equation (NLSE) with constant parameters. However, in realistic situation, the governing equation should be extended to the vector coupled NLSE with space-dependent coefficients, this is because the characteristic coefficients of realistic birefringent fibers are often spatially inhomogeneous [7-10]. Hence, this paper focuses on the nonlinear dynamics of Thirring optical solitons in spatially inhomogeneous birefringent fibers.

Under investigation in this work is the following vector coupled NLSE with space-modulated coefficients:

$$iq_t + a_1(x)q_{xx} + b_1(x)q_{xt} + c_1(x)|r|^2 q = 0 \qquad (1)$$

$$ir_t + a_2(x)r_{xx} + b_2(x)r_{xt} + c_2(x)|q|^2 r = 0 \qquad (2)$$

where $q(x,t)$ and $r(x,t)$ are complex-valued wave profiles of two split pulses that are the functions of variables x and t, the subscripts denote the partial derivatives with respect to corresponding variables. For $l = 1, 2$, $a_l(x)$, $b_l(x)$ and $c_1(x)$ represent the GVD, spatio-temporal dispersion (STD) and XPM that are space-modulated coefficients. It's worth noting that the dynamical model that includes both GVD and STD is well posed [1]. Additionally, the first term, in Eqs. (1) and (2), represents the time evolution term.

It should be noted that explicit Thirring optical solitons in homogeneous birefringent fibers with Kerr law nonlinearity and STD (i.e. $a_l(x)$, $b_l(x)$ and $c_1(x)$ in Eqs. (1) and (2) are real constants) were reported earlier [1]. However, to our knowledge, analytical Thirring optical 1-solitons to Eqs. (1) and (2) have not been derived. Then, this paper is thus an extension of our previous results. As a consequence, via the ansatz method [11-42], exact Thirring optical solitons to the well-posed model (1) and (2) are obtained along with their respective constraint conditions.

2. Thirring bright solitons

For Thirring bright (bell) solitons, we first introduce the following hypothesis:

$$q(x,t) = A_1(x)\,\mathrm{sech}[B_1(x)x]\exp(-i\delta_1 t) \qquad (3)$$

$$r(x,t) = A_2(x)\,\mathrm{sech}[B_2(x)x]\exp(-i\delta_2 t) \qquad (4)$$

In Eqs. (3) and (4), for $l = 1, 2$, $A_l(x)$ and $B_l(x)$ represent the amplitudes and inverse widths of two polarized solitons, and δ_l is a real constant.

Substituting Eqs. (3) and (4) into Eqs. (1) and (2) gives

$$\begin{aligned}
&\frac{d^2A_l(x)}{dx^2}\mathrm{sech}[B_l(x)x] + A_l(x)[\frac{d(B_l(x)x)}{dx}]^2\mathrm{sech}[B_l(x)x] \\
&-2\frac{dA_l(x)}{dx}\frac{d(B_l(x)x)}{dx}\mathrm{sech}[B_l(x)x]\tanh[B_l(x)x] \\
&-A_l(x)\frac{d^2(B_l(x)x)}{dx^2}\mathrm{sech}[B_l(x)x]\tanh[B_l(x)x] \\
&-2A_l(x)[\frac{d(B_l(x)x)}{dx}]^2\mathrm{sech}^3[B_l(x)x] - \frac{i\delta_l b_l(x)}{a_l(x)}\frac{dA_l(x)}{dx}\mathrm{sech}[B_l(x)x] \\
&+\frac{i\delta_l b_l(x)A_l(x)}{a_l(x)}\frac{d(B_l(x)x)}{dx}\mathrm{sech}[B_l(x)x]\tanh[B_l(x)x] \\
&+\frac{\delta_l}{a_l(x)}A_l(x)\,\mathrm{sech}[B_l(x)x] + \frac{c_l(x)}{a_l(x)}A_l(x)A_{\bar{l}}^2(x)\,\mathrm{sech}^2[B_{\bar{l}}(x)x]\mathrm{sech}[B_l(x)x] = 0
\end{aligned} \qquad (5)$$

for $l = 1, 2$ and $\bar{l} = 3 - l$.

Applying the balancing principle gives

$$B_{\bar{l}}(x) = B_l(x) = B(x) \tag{6}$$

Then, setting the coefficients of $\operatorname{sech}^i[B(x)x]\,\operatorname{tann}^j[B(x)x]$ ($i = 1, 3,\ j = 0,1$) to zero yields

$$\begin{aligned} &\operatorname{sech}^1[B(x)x]: \\ &\frac{d^2A_l(x)}{dx^2} + A_l(x)[\frac{d(B(x)x)}{dx}]^2 \\ &-\frac{i\delta_l b_l(x)}{a_l(x)}\frac{dA_l(x)}{dx} + \frac{\delta_l}{a_l(x)}A_l(x) = 0 \end{aligned} \tag{7}$$

$$\begin{aligned} &\operatorname{sech}^1[B(x)x]\tanh^1[B(x)x]: \\ &-2\frac{dA_l(x)}{dx}\frac{d(B(x)x)}{dx} - A_l(x)\frac{d^2(B(x)x)}{dx^2} \\ &+\frac{i\delta_l b_l(x)}{a_l(x)}A_l(x)\frac{d(B(x)x)}{dx} = 0 \end{aligned} \tag{8}$$

$$\begin{aligned} &\operatorname{sech}^3[B(x)x]: -2A_l(x)[\frac{d(B(x)x)}{dx}]^2 \\ &+\frac{c_l(x)}{a_l(x)}A_l(x)A_{\bar{l}}^2(x) = 0 \end{aligned} \tag{9}$$

Solving the above system of equations, we obtain

$$\begin{aligned} b_l(x) = &-2i\frac{a_l(x)}{\delta_l A_l(x)}\frac{dA_l(x)}{dx} \\ &-\frac{ia_l(x)}{\delta_l}\left(\frac{d^2(B(x)x)}{dx^2}\Big/\frac{d(B(x)x)}{dx}\right) \end{aligned} \tag{10}$$

$$c_l(x) = 2a_l(x)[\frac{1}{A_{\bar{l}}(x)}\frac{d(B(x)x)}{dx}]^2 \tag{11}$$

$$\begin{aligned} &\frac{d^2A_l(x)}{dx^2} + A_l(x)[\frac{d(B(x)x)}{dx}]^2 + \frac{\delta_l}{a_l(x)}A_l(x) = \\ &\frac{2}{A_l(x)}\left[\frac{dA_l(x)}{dx}\right]^2 + \frac{dA_l(x)}{dx}\left(\frac{d^2(B(x)x)}{dx^2}\Big/\frac{d(B(x)x)}{dx}\right) \end{aligned} \tag{12}$$

where $A_l(x)$ and $B(x)$ are arbitrary functions, and δ_l is a arbitrary constant.

Hence, exact Thirring bright solitons in homogeneous birefringent fibers with Kerr law nonlinearity and STD are obtained, which are given by Eqs. (3) and (4). Additionally, the coefficient constraint conditions that need to hold for the existence of those Thirring bright optical solitons are given by Eqs. (10)-(12), in which Eq. (10) gives the STD coefficient and Eq. (11) gives the XPM coefficient.

3. Thirring dark solitons

For Thirring dark (kink) solitons, we first introduce the following hypothesis:

$$q(x,t) = A_1(x)\tanh[B_1(x)x]\exp(-i\delta_1 t) \tag{13}$$

$$r(x,t) = A_2(x)\tanh[B_2(x)x]\exp(-i\delta_2 t) \tag{14}$$

Substituting Eqs. (13) and (14) into Eqs. (1) and (2) gives

$$\begin{aligned} &\frac{d^2A_l(x)}{dx^2}\tanh[B_l(x)x] + 2\frac{dA_l(x)}{dx}\frac{d(B_l(x)x)}{dx} \\ &-2\frac{dA_l(x)}{dx}\frac{d(B_l(x)x)}{dx}\tanh^2[B_l(x)x] \\ &+A_l(x)\frac{d^2(B_l(x)x)}{dx^2} - A_l(x)\frac{d^2(B_l(x)x)}{dx^2}\tanh^2[B_l(x)x] \\ &-2A_l(x)[\frac{d(B_l(x)x)}{dx}]^2\tanh[B_l(x)x] + \frac{\delta_l}{a_l(x)}A_l(x)\tanh[B_l(x)x] \\ &+2A_l(x)[\frac{d(B_l(x)x)}{dx}]^2\tanh^3[B_l(x)x] \\ &-\frac{i\delta_l b_l(x)}{a_l(x)}\frac{dA_l(x)}{dx}\tanh[B_l(x)x] - \frac{i\delta_l b_l(x)}{a_l(x)}A_l(x)\frac{d(B_l(x)x)}{dx} \\ &+\frac{i\delta_l b_l(x)}{a_l(x)}A_l(x)\frac{d(B_l(x)x)}{dx}\tanh^2[B_l(x)x] \\ &+\frac{c_l(x)}{a_l(x)}A_l(x)A_{\bar{l}}^2(x)\tanh^2[B_{\bar{l}}(x)x]\tanh[B_l(x)x] = 0 \end{aligned} \tag{15}$$

The balancing principle again yields Eq. (6). Then, equating the coefcient of $\operatorname{tann}^j[B(x)x]$ ($j = 0, 1, 2, 3$) to zero yields

$$\begin{aligned} &\operatorname{tann}^0[B(x)x]: \\ &2\frac{dA_l(x)}{dx}\frac{d(B(x)x)}{dx} + A_l(x)\frac{d^2(B(x)x)}{dx^2} \\ &-\frac{i\delta_l b_l(x)}{a_l(x)}A_l(x)\frac{d(B(x)x)}{dx} = 0 \end{aligned} \tag{16}$$

$$\begin{aligned} &\tanh^1[B(x)x]: \\ &\frac{d^2A_l(x)}{dx^2} - 2A_l(x)[\frac{d(B(x)x)}{dx}]^2 \\ &-\frac{i\delta_l b_l(x)}{a_l(x)}\frac{dA_l(x)}{dx} + \frac{\delta_l}{a_l(x)}A_l(x) = 0 \end{aligned} \tag{17}$$

$$\begin{aligned} &\tanh^2[B(x)x]: \\ &-2\frac{dA_l(x)}{dx}\frac{d(B(x)x)}{dx} - A_l(x)\frac{d^2(B(x)x)}{dx^2} \\ &+\frac{i\delta_l b_l(x)}{a_l(x)}A_l(x)\frac{d(B(x)x)}{dx} = 0 \end{aligned} \tag{18}$$

$$\tanh^3[B(x)x]:$$
$$2A_l(x)[\frac{d(B(x)x)}{dx}]^2+\frac{c_l(x)}{a_l(x)}A_l(x)A_{\bar{l}}^2(x)=0 \tag{19}$$

Solving the above system of equations, we obtain

$$\begin{aligned} b_l(x)&=-2i\frac{a_l(x)}{\delta_l A_l(x)}\frac{dA_l(x)}{dx}\\ &-\frac{ia_l(x)}{\delta_l}\left(\frac{d^2(B(x)x)}{dx^2}\middle/\frac{d(B(x)x)}{dx}\right)\end{aligned} \tag{20}$$

$$c_l(x)=-2a_l(x)[\frac{1}{A_{\bar{l}}(x)}\frac{d(B(x)x)}{dx}]^2 \tag{21}$$

$$\begin{aligned}&\frac{d^2A_l(x)}{dx^2}-2A_l(x)[\frac{d(B(x)x)}{dx}]^2+\frac{\delta_l}{a_l(x)}A_l(x)=\\ &\frac{2}{A_l(x)}\left[\frac{dA_l(x)}{dx}\right]^2+\frac{dA_l(x)}{dx}\left(\frac{d^2(B(x)x)}{dx^2}\middle/\frac{d(B(x)x)}{dx}\right)\end{aligned} \tag{22}$$

Eq. (10) gives the STD coefficient and Eq. (11) gives the XPM coefficient.

Finally, explicit Thirring darksolitons in homogeneous birefringent fibers with Kerr law nonlinearity and STD are constructed, which are given by Eqs. (13) and (14). Additionally, the constraints for the existence of those Thirring dark solitons are derived, which are given by Eqs. (20)-(22).

4. Thirring singular solitons

For Thirring singular solitons, we first introduce the following hypothesis:

$$q(x,t)=A_1(x)\coth[B_1(x)x]\exp(-i\delta_1 t) \tag{23}$$

$$r(x,t)=A_2(x)\coth[B_2(x)x]\exp(-i\delta_2 t) \tag{24}$$

Substituting Eqs. (23) and (24) into Eqs. (1) and (2) gives

$$\begin{aligned}&\frac{d^2A_l(x)}{dx^2}\coth[B_l(x)x]-2\frac{dA_l(x)}{dx}\frac{d(B_l(x)x)}{dx}\coth^2[B_l(x)x]\\ &-A_l(x)\frac{d^2(B_l(x)x)}{dx^2}\coth^2[B_l(x)x]+2A_l(x)[\frac{d(B_l(x)x)}{dx}]^2\coth^3[B_l(x)x]\\ &-2A_l(x)[\frac{d(B_l(x)x)}{dx}]^2\coth[B_l(x)x]+2\frac{dA_l(x)}{dx}\frac{d(B_l(x)x)}{dx}+A_l(x)\frac{d^2(B_l(x)x)}{dx^2}\\ &-i\frac{\delta_l b_l(x)}{a_l(x)}\frac{dA_l(x)}{dx}\coth[B_l(x)x]+i\frac{\delta_l b_l(x)}{a_l(x)}A_l(x)\frac{d(B_l(x)x)}{dx}\coth^2[B_l(x)x]\\ &-i\frac{\delta_l b_l(x)}{a_l(x)}A_l(x)\frac{d(B_l(x)x)}{dx}+\frac{\delta_l}{a_l(x)}A_l(x)\coth[B_l(x)x]\\ &+\frac{c_l(x)}{a_l(x)}A_l(x)A_{\bar{l}}^2(x)\coth^2[B_{\bar{l}}(x)x]\coth[B_l(x)x]=0\end{aligned} \tag{25}$$

The balancing principle again yields Eq. (6). Then, equating the coefcient of $\coth^j[B(x)x]$ ($j=0,1,2,3$) to zero gives

$$\coth^0[B(x)x]:$$
$$\begin{aligned}&2\frac{dA_l(x)}{dx}\frac{d(B(x)x)}{dx}+A_l(x)\frac{d^2(B(x)x)}{dx^2}\\ &-i\frac{\delta_l b_l(x)}{a_l(x)}A_l(x)\frac{d(B(x)x)}{dx}=0\end{aligned} \tag{26}$$

$$\coth[B(x)x]:$$
$$\begin{aligned}&\frac{d^2A_l(x)}{dx^2}-2A_l(x)[\frac{d(B(x)x)}{dx}]^2\\ &-i\frac{\delta_l b_l(x)}{a_l(x)}\frac{dA_l(x)}{dx}+\frac{\delta_l}{a_l(x)}A_l(x)=0\end{aligned} \tag{27}$$

$$\coth^2[B(x)x]:$$
$$\begin{aligned}&-2\frac{dA_l(x)}{dx}\frac{d(B(x)x)}{dx}-A_l(x)\frac{d^2(B(x)x)}{dx^2}\\ &+i\frac{\delta_l b_l(x)}{a_l(x)}A_l(x)\frac{d(B(x)x)}{dx}=0\end{aligned} \tag{28}$$

$$\coth^3[B(x)x]:$$
$$2A_l(x)[\frac{d(B(x)x)}{dx}]^2+\frac{c_l(x)}{a_l(x)}A_l(x)A_{\bar{l}}^2(x)=0 \tag{29}$$

Solving the above system of equations, we can get the constraint conditions for the existence of Thirring singular solitons, which are same as Eqs. (20)-(22).

Finally, analytical Thirring singular solitons in homogeneous birefringent fibers with Kerr law nonlinearity and STD are derived, which are given by Eqs. (23) and (24).

5. Conclusion

The vector coupled NLSE with space-modulated coefficients, describing the propagation of Thirring optical solitons in spatially inhomogeneous birefringent fibers, has been investigated analytically. The Kerr law nonlinearity and spatio-temporal dispersion are taken into account. Based on the ansatz method, exact Thirring bright, dark and singular 1-solitons are obtained. Additionally, the coefficient restrictions for the existence of those Thirring solitons are reported.

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