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Analysis of complex refractive index solid-core Bragg fibers

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We investigate the modal characteristics of solid-core Bragg fibers with complex refractive index profiles using Galerkin's method under the scalar linearly polarized (LP) mode approximation. The imaginary part of the electric field results in wave-front distortion which is critically dependent on the imaginary part of the refractive index. The optical gain is calculated for three physically acceptable values of the imaginary part of the refractive index. As the operation wavelength increases, the gain decreases because the optical power confined to the core decreases.

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1. Introduction

Bragg fibers are light guides that consist of a low-index central region (serving as the core) that is surrounded by concentric layers of alternate high and low refractive index materials. Although in the first proposal [1] the refractive indices of the cladding bilayers were assumed to be higher than that of the core, Bragg fibers could also be designed such that only one of the cladding bilayers has a refractive index higher than that of the core [2]. In either case, the refractive index periodicity in the cladding spawns a photonic bandgap and thus, if the frequency of the incident light falls within the bandgap, it can be confined within the core of this fiber [3].

Bragg fibers have attracted increasing attention due mainly to their large photonic bandgaps and omnidirectional reflectivity [4]. Rapid progress in infrared energy transmission [5], external reflection [6], and optical detection [7] has been made in recent years, with stress on large transmission bandwidth and low propagation loss. Reduction of propagation loss is of primary interest since in future Bragg fiber laser and amplifier applications, background loss will severely limit the net gain. The first report on a solid core Bragg fiber appeared in 2000 [8] which aim to achieve zero group velocity dispersion (GVD) at λ=1.06 μm corresponding to the emission wavelength of Yb^{+3}- doped fiber lasers. A study of non-linear pulse propagation in such fibers has recently been reported in the literature [9].

The main characteristic in a fiber laser or amplifier is the gain along the length of the fiber. Except for models based on rate and propagation equations, an alternative method is to analyze optical fibers whose refractive-index profile is described in terms of a complex function. Then, the gain can be described by the imaginary component of the complex propagation constant, which is critically dependent on the imaginary component of the complex refractive-index profile. Some approximate or numerical methods have been presented [10-12] for evaluation of the propagation characteristics of such fibers. Moreover, it has been shown that the use of ytterbium-doped photonic bandgap fibers as fiber-lasers or amplifiers lead to improved features of amplification properties with respect to standard step-index fibers [13].

In this work, we use a scalar approximation to the Helmholtz equation based on Galerkin's method to investigate the guidance of optical waves in solid-core Bragg fibers with complex refractive index profiles (RIP). The modal properties are discussed and a noticeable phenomenon of wave-front distortion is demonstrated. The spectral characteristics of gain are also studied for three physically acceptable values of the imaginary part of the RIP.

2. Mathematical model and numerical approach

Under the weakly guiding approximation, the scalar Helmholtz eigenvalue equation, for a given azimuthal mode number *m,* can be written in polar coordinates as [14]

$$\frac{d^2E(r)}{dr^2}+\frac{1}{r}\frac{dE(r)}{dr}+\left(k^2n^2(r)-\beta^2-\frac{m^2}{r^2}\right)E(r)=0 \qquad (1)$$

where E=$E(r)$ represents the radial variation of the modal field, k is the free-space wavenumber and β is the propagation constant. The scalar wave-equation (1) is known to be a good approximation to the full-vectorial treatment for microstructures with small refractive index-contrast, which is what we consider here. The 'real' RIP $n_r(r)$ of a solid-core Bragg fiber is expressed by a staircase function:

$$n_r(r)=\begin{cases} n_{co}, & r \le r_{co} \\ n_H, & r = r_{m,a} \\ n_L, & r = r_{m,b} \end{cases} \tag{2}$$

with

$$\begin{aligned} r_{a,m} &= r-[r_{co}+(m-1)\Lambda], \quad 0 \le r_{a,m} \le a \text{ for layers with thickness } a \\ r_{b,m} &= r-[r_{co}+(m-1)\Lambda+a],\ 0 \le r_{b,m} \le b \text{ for layers with thickness } b \end{aligned} \tag{3}$$

where r_{co} the core radius, a and b the up-doped and down-doped layer thickness, respectively, $\Lambda= a +b$ the radial multilayer period and m=1,2,3…

In order to investigate a structure with gain, we assume, instead of Eq.(2), a complex form of the RIP:

$$n(r)=n_r(r)+in_i(r) \tag{4}$$

The imaginary part n_i of RIP can be considered as a function of pump/signal wavelength and dopant profiles of active material [10]. For simplicity, we assume a constant n_i throughout this paper. Consequently, the scalar Helmholtz eigenvalue equation becomes non-Hermitian, meaning the eigenvalues may be complex. We thus expect the propagation constant to have both real and imaginary part, writing $\beta= \beta r+i\beta i$. If βi is positive the solutions will decay exponentially corresponding to loss in optical power, whereas a negative value of βi results in solutions increasing in power along the fiber length, corresponding to gain.

We apply Galerkin's numerical method in cylindrical coordinates [15, 16] to find the modal field of the Bragg fiber with complex refractive index profile. The complex modal field is expanded in a set of associated Laguerre-Gauss basis functions

$$E(R)=\sum_{i=0}^{N-1} a_i\, \varphi_i^m(R) \tag{5}$$

where

$$\varphi_i^m(R)=\sqrt{\frac{i!}{(i+m)!}}\, e^{-m/2}e^{-R^2/2}\, L_i^m(R), \quad i=0,1,...N-1 \tag{6}$$

$L_i^m(R)$ are the associated Laguerre polynomial and N is the number of basis functions used. The normalized parameter R is defined as $R=\sigma r^2/\alpha^2$ where α is the core radius and σ is an arbitrary positive number that affects the convergence, accuracy and computational time [16]. Then, Eq.(1) is transformed into a matrix eigenvalue equation for the propagation constant as

$$[H][A]=\beta^2[A] \tag{7}$$

where the elements of the complex $N\times N$ matrix H, are given by

$$H_{ij}=-\frac{\sigma}{\alpha^2}T_{ij}+k^2 I_{ij} \tag{8}$$

where

$$T_{ij}=(2j+m+1)\delta_{ij}+\sqrt{(j+1)(j+m+1)}\,\delta_{j+1,i}+\sqrt{j\,(j+m)}\,\delta_{j-1,i} \tag{9}$$

and

$$I_{ij}=\int_0^\infty n^2\left(\sqrt{R/\sigma}\right)\varphi_i^m\,\varphi_j^m\,R\,dR \tag{10}$$

The eigenvector $[A]$ is a column vector that contains the complex coefficients a_i. In our calculations, the parameter σ is chosen equal to core radius α of the Bragg fiber and the complex eigenvalue problem is solved using optimized EISPACK matrix eigensystem routines [17]. As a final result, the complex propagation constant β of a guided mode is obtained and the gain of the propagating signal power, in decibels per meter, is given by g (*dB/m*) =20 *log* (*e*) *Im*(β).

3. Modal characteristics of solid-core Bragg fibers with gain

The design we have investigated is a depressed index solid-core Bragg fiber [8] having a core radius r_{co}= 6.7 μm with refractive index n_{co}=1.446, and a periodic cladding with layers of thicknesses a = 2b=1.2 μm having refractive indices n_H = 1.459, n_L = 1.450 (at 1.06 μm), respectively. Justification for the choice of these parameters is given in Ref. [18]. This fiber has a unique mode LP_{01} guided at this wavelength with zero GVD. The computed electric field of this unique mode LP_{01} together with the 'real' RIP of the considered fiber is plotted in Fig. 1. As it is expected, the modal field is zero at the core/cladding interface and at the transitions from low to high refractive index region and decays with the radial distance similar to a 'sinc' function.

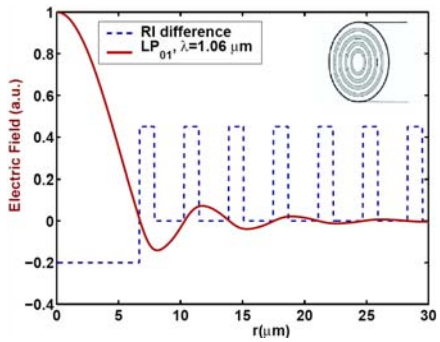
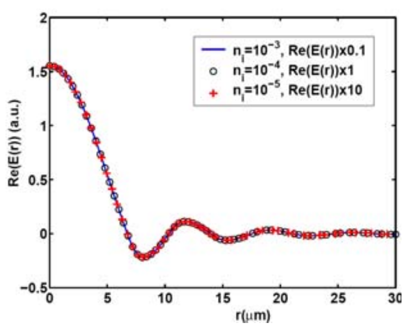
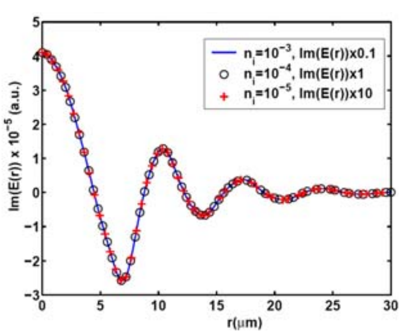


Fig.1. Refractive index profile and computed electric field of real refractive index solid-core Bragg fiber. The field has been normalized to its maximum.

Fig. 2 (a) and (b) show the real and imaginary part of the modal field LP_{01}, respectively, for three physically acceptable values of the imaginary part n_i at wavelength λ=1.06 μm. As it is seen, the imaginary part of the modal electric field is much smaller than the real part because of the small n_i. Furthermore, the imaginary parts of the modal fields in Fig. 2 (b) are overlapped after multiplication by 10 for $n_i=10^{-4}$ and by 100 for $n_i=10^{-5}$. An apparent characteristic of the imaginary part of the modal field is that the field reaches its minimum near the edge of the fiber core. Moreover, both real and imaginary parts of the modal field reflect the symmetry of the periodic multilayer structure.



(a)



(b)

Fig. 2. Real part (a) and Imaginary part (b) of the modal field LP_{01} of complex refractive index Bragg fiber at wavelength λ=1.06 μm.

Fig. 3 shows the phase distribution in the fiber's cross section for the three different values of n_i at λ=1.06 μm. This phase can be considered to be the wave-front of the fundamental mode LP_{01}. It is well known that the wave-front is a plane for fibers without gain, but when an imaginary part of the refractive index is introduced, the wave-front is distorted. We found that the phase distortion is principally dominated by the imaginary part of the refractive index n_i. Moreover, as it is expected, phase-singularities are observed at the core/cladding interface and at the transitions from low to high refractive index region.

Fig. 4 shows the gain in dB/m, as a function of wavelength for the three different values of n_i. As it is seen, when the gain corresponding to $n_i=10^{-3}$ is divided by 10 and the gain corresponding to $n_i=10^{-5}$ multiplied by 10, the curves are overlapped and therefore the gain is analogue to n_i. Furthermore, the gain decreases as the operating wavelength increases because the optical power confined to the core decreases as the wavelength increases [19].

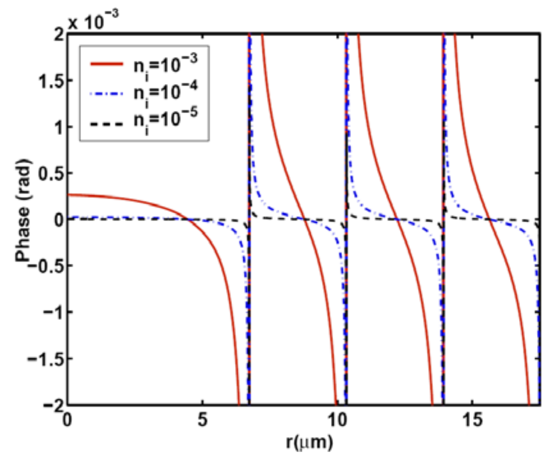


Fig. 3. Radial phase distribution at λ=1.06 μm for three different values of n_i.

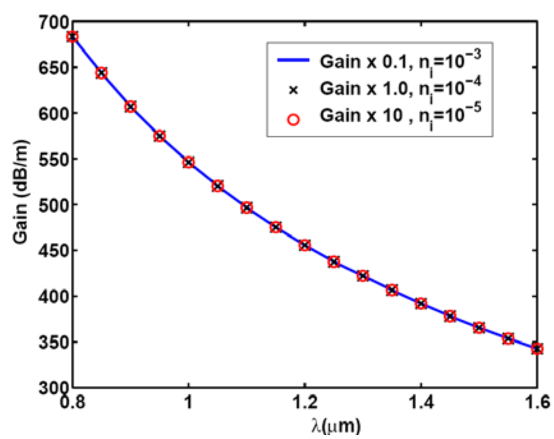


Fig. 4. Gain versus operating wavelength. The gain has been divided by 10 for $n_i=10^{-3}$ and multiplied by 10 for $n_i=10^{-5}$ so that the gain-curves for the three values of n_i are overlapped.

4. Conclusions

We investigated the propagation characteristics of single-mode solid-core Bragg fibers with complex refractive index profiles applying Gelerkin's method under the well-known scalar linearly a polarized (LP) mode approximation. The gain is calculated by the imaginary component of the complex propagation constant, which is critically dependent on the imaginary component of the complex refractive-index profile. The imaginary part of the electric field results in wave-front distortion of the propagating mode in the Bragg fiber with gain. It can be assumed that the modal field is a combination of some normal modes of the corresponding fiber without gain and therefore, induces a wave-front distortion. The optical gain is calculated for three physically acceptable values of the imaginary part of the refractive index and its wavelength dependence is examined. As the operating wavelength λ increases, the gain decreases because the optical power confined to the core decreases.

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