

# Analysis of buffered multilayer optical planar waveguides

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A variational method is used to study the effects of addition of a low-index buffer layer over a high-index substrate on the propagation characteristics of a waveguide. For a buffered leaky waveguide with a single mode, the TM mode is more leaky out into the high-index substrate in comparison with the TE mode. Also, for a buffered leaky waveguide with three modes, the TE<sub>2</sub> (TM<sub>2</sub>) mode is more leaky out and with a faster oscillation into the high-index substrate in comparison with the TE<sub>1</sub> (TM<sub>1</sub>) and TE<sub>0</sub> (TM<sub>0</sub>) modes.

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## 1. Introduction

The knowledge of optical and electrical propagation properties of multilayer planar waveguides is very important for engineering design of guided-wave optoelectronic devices [1-5]. Thus, the effect of a buffer material with a lower refractive index between the waveguiding structure and the high-index low-cost silicon substrate were studied by using a finite element method leaky mode solver [1].

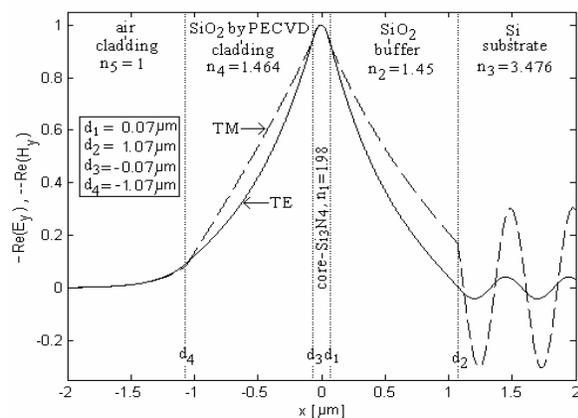


Fig. 1. The real part of the fundamental field profiles ( $E_y$ ,  $H_y$ ) of the buffered leaky waveguide: TE (-) and TM (- -) modes. The field amplitude has been normalized to a maximum value of unity.

In this paper we apply a variational method for exact location of each of the zeros of the dispersion equation which correspond to the leaky modes of waveguides, by using of the exact analytical eigenfunctions [2].

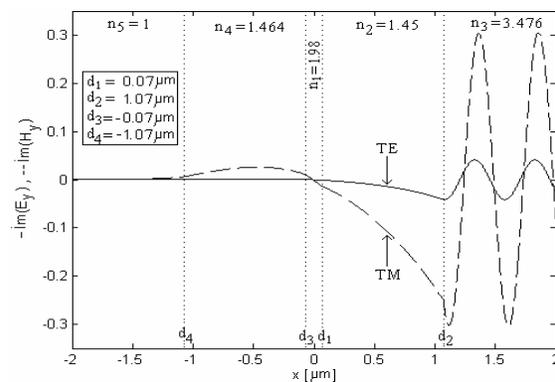


Fig. 2. The imaginary part of the fundamental field profiles ( $E_y$ ,  $H_y$ ) of the buffered leaky waveguide: TE (-) and TM (- -) modes.

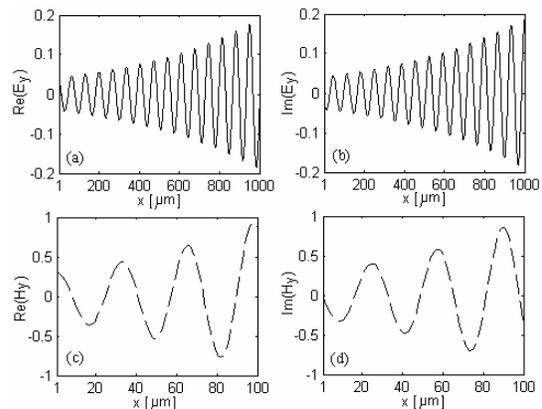


Fig. 3. Detail of the real and imaginary parts of the fundamental field profiles ( $E_y$ ,  $H_y$ ) of the buffered leaky waveguide ( $d_1 = 0.07 \mu\text{m}$ ,  $d_2 = 1.07 \mu\text{m}$ ,  $d_3 = -0.07 \mu\text{m}$ ,  $d_4 = -1.07 \mu\text{m}$ ,  $n_1 = 1.98$ ,  $n_2 = 1.45$ ,  $n_3 = 3.476$ ,  $n_4 = 1.464$ ,  $n_5 = 1$ ,  $\lambda = 1.55 \mu\text{m}$ ) for a large thickness of the substrate: TE (-) and TM (- -) modes.

The buffered leaky planar waveguides are obtained by placing a buffer material with a lower refractive index between the waveguiding structure and the high-index low-cost silicon substrate. The scalar-wave equation for a buffered leaky planar waveguide (Fig. 1) is given by

$$\frac{d^2\psi(x)}{dx^2} + k^2 n^2(x)\psi(x) = \beta^2\psi(x), \quad (1)$$

where  $\beta$  is the propagation constant,  $k$  is the free space wave number,  $n(x)$  is the refractive index profile

$$n(x) = \begin{cases} n_1, & \text{for } d_3 \leq x \leq d_1, d_3 = -d_1, d_1 > 0, \\ n_2, & \text{for } d_1 < x \leq d_2, d_2 > 0, \\ n_3, & \text{for } d_2 < x < \infty, \\ n_4, & \text{for } d_4 < x < d_3, d_4 < 0, \\ n_5, & \text{for } -\infty < x < d_4, \end{cases} \quad (2)$$

$n_1, n_2, n_3, n_4$  and  $n_5$  are the refractive indices of the core, SiO<sub>2</sub> buffer, Si substrate, SiO<sub>2</sub> by PECVD cladding and air cladding, respectively ( $n_3 > n_1 > n_4 > n_2 > n_5$ ),  $d_1 - d_3, d_2 - d_1$ , semi-infinite,  $d_3 - d_4$ , and semi-infinite are the thickness of these layers, respectively. The effective index  $\beta/k$  for the TE and TM modes can be found from the dispersion equation which is obtained by applying the boundary conditions at the interfaces between different layers.

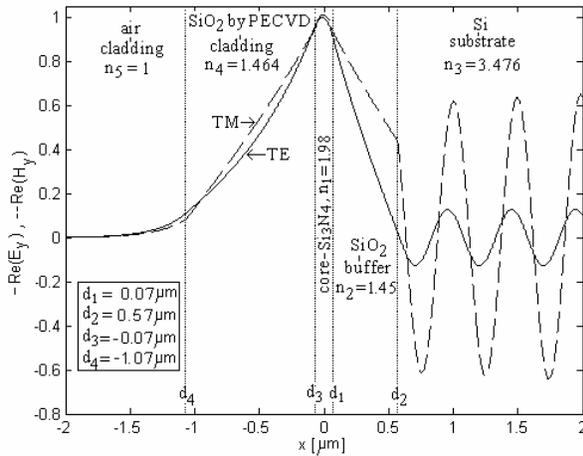


Fig.4. The real part of the fundamental field profiles ( $E_y, H_y$ ) of the buffered leaky waveguide: TE (-) and TM (- -) modes. The field amplitude has been normalized to a maximum value of unity.

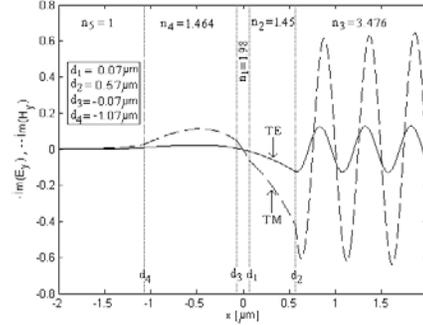


Fig.5. The imaginary part of the fundamental field profiles ( $E_y, H_y$ ) of the buffered leaky waveguide: TE (-) and TM (- -) modes.

## 2. Numerical results and conclusions

We have calculated the exact value of the effective index  $\beta/k$  for a waveguide with  $d_1 = 0.07\mu\text{m}$ ,  $d_2 = 1.07\mu\text{m}$ ,  $d_3 = -0.07\mu\text{m}$ ,  $d_4 = -1.07\mu\text{m}$ ,  $n_1 = 1.98$ ,  $n_2 = 1.45$ ,  $n_3 = 3.476$ ,  $n_4 = 1.464$ ,  $n_5 = 1$ ,  $\lambda = 1.55\mu\text{m}$  (Table 1).

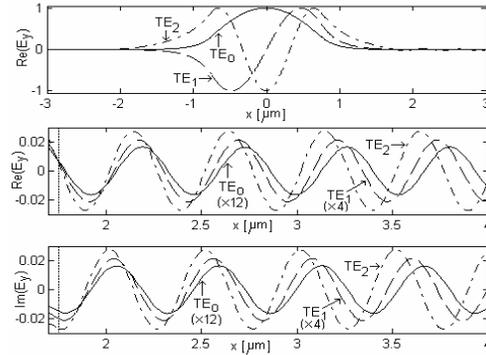


Fig.6. Detail for the real and imaginary parts of the first three mode field profiles ( $E_y$ ) of the buffered leaky waveguide ( $d_1 = 0.75\mu\text{m}$ ,  $d_2 = 1.75\mu\text{m}$ ,  $d_3 = -0.75\mu\text{m}$ ,  $d_4 = -1.75\mu\text{m}$ ): TE<sub>0</sub> (-), TE<sub>1</sub> (- -) and TE<sub>2</sub> (- . -).

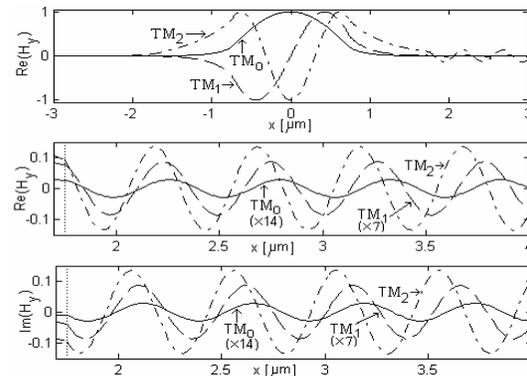


Fig.7. Detail for the real and imaginary parts of the first three mode field profiles ( $H_y$ ) of the buffered leaky waveguide ( $d_1 = 0.75\mu\text{m}$ ,  $d_2 = 1.75\mu\text{m}$ ,  $d_3 = -0.75\mu\text{m}$ ,

$d_4 = -1.75\mu\text{m}$ ):  $TM_0$  (-),  $TM_1$  (- -) and  $ME_2$  (- . -).

The results are in agreement with the previously published values [1]. The real and imaginary parts of the TE and TM weakly mode profiles of this structure, for buffer layer thickness of  $1\mu\text{m}$  and the thickness of the guiding film of  $0.14\mu\text{m}$ , are shown at the same scale (each field amplitude has been normalized to a maximum value of unity) in Fig. 1 and in Fig.2, respectively. The TM mode is more leaky out into the high-index substrate in

comparison with the TE mode. The leaky modes oscillate periodically and increase in the transverse direction of the substrate at infinity. These increases for the real and imaginary parts of the field profiles are so small in our example that they cannot be shown at the scale of Figs. 1-2. Fig. 3 shows the detail of the real and imaginary parts of the fundamental field profiles ( $E_y$ ,  $H_y$ ) of the buffered leaky waveguide for a large thickness of the substrate.

Table 1. Comparison of the propagation constant  $\beta$  and the effective index  $\beta/k$  for two waveguides which differ by the buffer thicknesses: ( $d_1 = 0.07\mu\text{m}$ ,  $d_2 = 0.57\mu\text{m}$ ,  $d_3 = -0.07\mu\text{m}$ ,  $d_4 = -1.07\mu\text{m}$ ,  $n_1 = 1.98$ ,  $n_2 = 1.45$ ,  $n_3 = 3.476$ ,  $n_4 = 1.464$ ,  $n_5 = 1$ ,  $\lambda = 1.55\mu\text{m}$ ) and ( $d_1 = 0.07\mu\text{m}$ ,  $d_2 = 1.07\mu\text{m}$ ,  $d_3 = -0.07\mu\text{m}$ ,  $d_4 = -1.07\mu\text{m}$ ,  $n_1 = 1.98$ ,  $n_2 = 1.45$ ,  $n_3 = 3.476$ ,  $n_4 = 1.464$ ,  $n_5 = 1$ ,  $\lambda = 1.55\mu\text{m}$ ).

Mode	$d_2 - d_1 = 0.5\mu\text{m}$ , $d_1 - d_3 = 0.14\mu\text{m}$ , $\beta$ $\beta/k$	$d_2 - d_1 = 1\mu\text{m}$ , $d_1 - d_3 = 0.14\mu\text{m}$ , $\beta$ $\beta/k$
TE <sub>0</sub>	6.1180936100 – 0.0309326605j 1.5092735025 – 0.0076307830j	6.1892610225 – 0.0031220002j 1.5268298030 – 0.0007701667j
TM <sub>0</sub>	5.9631474993 – 0.1056182363j 1.4710498214 – 0.0260549798j	5.9765145378 – 0.0248861721j 1.4743473383 – 0.0061391738j

Table 2. Comparison of the propagation constant  $\beta$  and the effective index  $\beta/k$  for a buffered leaky planar waveguide with a larger film thickness ( $d_1 = 0.75\mu\text{m}$ ,  $d_2 = 1.75\mu\text{m}$ ,  $d_3 = -0.75\mu\text{m}$ ,  $d_4 = -1.75\mu\text{m}$ ,  $n_1 = 1.98$ ,  $n_2 = 1.45$ ,  $n_3 = 3.476$ ,  $n_4 = 1.464$ ,  $n_5 = 1$ ,  $\lambda = 1.55\mu\text{m}$ ) for TE and TM modes.

Mode	TE, $d_2 - d_1 = 1\mu\text{m}$ , $d_1 - d_3 = 1.5\mu\text{m}$ , $\beta$ $\beta/k$	TM, $d_2 - d_1 = 1\mu\text{m}$ , $d_1 - d_3 = 1.5\mu\text{m}$ , $\beta$ $\beta/k$
TE <sub>0</sub> TM <sub>0</sub>	7.8492591282 – 0.0000014864j 1.9363350043 – 0.0000003667j	7.8129578668 – 0.0000011643j 1.9273798402 – 0.0000002872j
TE <sub>1</sub> TM <sub>1</sub>	7.3099883158 – 0.0000238687j 1.8033022003 – 0.0000058882j	7.1762905051 – 0.0000439966j 1.7703202658 – 0.0000108535j
TE <sub>2</sub> TM <sub>2</sub>	6.4112902084 – 0.0006440885j 1.5816022188 – 0.0001588903j	6.2419135023 – 0.0042025760j 1.5398186518 – 0.0010367342j

Also, we have calculated the exact value of the effective index  $\beta/k$  for a waveguide ( $d_1 = 0.07\mu\text{m}$ ,  $d_2 = 0.57\mu\text{m}$ ,  $d_3 = -0.07\mu\text{m}$ ,  $d_4 = -1.07\mu\text{m}$ ,  $n_1 = 1.98$ ,  $n_2 = 1.45$ ,  $n_3 = 3.476$ ,  $n_4 = 1.464$ ,  $n_5 = 1$ ,  $\lambda = 1.55\mu\text{m}$ ) with a smaller buffer thickness (Table 1). The real and imaginary parts of the TE and TM weakly mode profiles of this structure, for buffer layer thickness of  $0.5\mu\text{m}$  and the thickness of the guiding film of  $0.14\mu\text{m}$ , are shown at the same scale in Fig.4 and in Fig.5, respectively. The TE and TM modes for this waveguide are more leaky out into the high-index substrate in comparison with the waveguide with a larger buffer thickness (compare with Figs 1-2).

Also, we have calculated the exact value of the effective index  $\beta/k$  for TE and TM modes of a buffered leaky planar waveguide ( $d_1 = 0.75\mu\text{m}$ ,  $d_2 = 1.75\mu\text{m}$ ,  $d_3 = -0.75\mu\text{m}$ ,  $d_4 = -1.75\mu\text{m}$ ,  $n_1 = 1.98$ ,  $n_2 = 1.45$ ,  $n_3 = 3.476$ ,  $n_4 = 1.464$ ,  $n_5 = 1$ ,  $\lambda = 1.55\mu\text{m}$ ) by using the first-order variational method with exact analytical eigenfunctions (Table 2). The real and imaginary parts of the TE<sub>0</sub> (TM<sub>0</sub>), TE<sub>1</sub> (TM<sub>1</sub>) and TE<sub>2</sub> (TM<sub>2</sub>) mode profiles of this structure for a buffer layer thickness ( $1\mu\text{m}$ ), and for a larger guiding film thickness ( $1.5\mu\text{m}$ ), are shown in Fig.6 (Fig.7). The TE<sub>2</sub> (TM<sub>2</sub>) mode is more leaky out and with a faster oscillation into the high-index substrate in comparison

with the TE<sub>1</sub>(TM<sub>1</sub>) and TE<sub>0</sub> (TM<sub>0</sub>) modes. We remember that the phase delay is proportional with the difference between the effective indexes of refraction and with the interaction length (see for example [5]). The phase velocity  $\omega/\beta$  of the TE<sub>0</sub> (TM<sub>0</sub>) mode is smaller than that of the TE<sub>1</sub> (TM<sub>1</sub>) and TE<sub>2</sub> (TM<sub>2</sub>) modes where  $\omega$  is the angular frequency.

Thus the variational method can be used for a better understanding of the effects of addition of a low-index buffer layer over a high substrate on the propagation characteristics of the waveguide.

## References

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