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An original method to compute the stresses in applied elasticity

E. OANTA*, A. NITA
Constantsa Maritime University, Constantsa , Romania

The paper presents an original method employed to compute the normal and tangential stresses generated by shear forces and bending moments. The method uses an algorithmic approach together with concepts from elasticity, analitical geometry, numerical methods and computer programming. A boolean algebra was created which uses simple shapes as basic elements employed in upper level operations. It was conceived an algorithm which offers the most relevant values of the stresses, based on a set of points located on the simple shaped bodies which are subjected to complex filtering conditions at a later stage. The method was implemented in an 12000 computer code lines application.

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1. Introduction

Every age of science is synchronuous with the calculation mean currently employed in science. The results of the research activity provide the theory and the concepts which are the basics for upper level research or industrial applications.

1.1 A historical perspective

Two centuries ago mathematicians used to organize calculation contests in order to find the best formulae and methods which offer results in a minimum amount of time. It was the age when Nicolo Tartaglia invented the relations later known as Cardano's formulae.

The industrial revolution needed new theoretical methods to be used in the design of the new structures made of new materials (cast iron, steel) and which had to support new types of loads, beside the mechanical loads. It was the age when the French mechanicists had important contributions.

The progresses in the nuclear and aerospace sciences required new means of calculation in order to perform very fast and accurate large amount of complex calculi with large amount of data. Computer science was asked to find the solutions and its progresses were an engine which offered a boost for all the sciences. New sciences appeared and new strategies of reserach were conceived.

In the actual conditions when the technical and scientific volume of information doubles every two years, computer became a common instrument in science.

1.2 Aims of the study

A legitimate question is why to use nowadays the applied elasticity theory when other methods, such as the finite element method is a common use and a useful method?

Applied elasticity offers several mathematical solutions which can be used to compute stresses, strains or deflections. The computer code which simply uses the classic methods is limited because of the basic simplifying hypotheses of the classic approach and because of the mathematical solution which is focused on a narrow class of problems.

Apart from these limitations, the strong point is that if the classic theory is employed together with other fields of science, applied elasticity may be used to create general analytic solutions which can be used either in dedicated models or in hybrid models.

The knowledge of the basic mathematical layer includes methods belonging to the theory of elasticity, analytical geometry, numerical methods, computer programming. The theory of elasticity offers general solutions from the classical point of view which can be used as deductive methods in finding particular solutions. Analytical geometry offers methods to parameterize the domains where the solutions are searched. If a boolean algebra is defined and simple geometrical bodies are considered as basic elements employed in upper level operations, most of the geometrical values used in applied elasticity may be easily computed. General numerical methods are a common instrument nowadays and the accuracy of the results may be easily predicted and controlled. This is why the general numerical methods are used in all the types of models: analytic, experimental (experimental data automatic processing) and dedicated numerical methods, such as FEM and FDM. Computer programming is one of the most important components of the strategy, because it connects all the other topics and it is synchronous with the information technology progress.

Coming back to the first sentence of the chapter, this strategy can be easily rejected by the people who base their judgments on their strong beliefs rather then exploring the new possibilities offered by the new interdisciplinary methods. Let us consider some of the advantages of the classic approaches:

- they are simple;
- there are no problems regarding the numerical stability of the solutions;
- accuracy is under control;
- the computing methods can be easily generalized using algorithmic approaches;
- if the original software application has a proper architecture, the libraries employed in applied elasticity models can be used in the upper level models, for dedicated industrial problems;
- they are inexpensive;
- they can be used to solve educational, design as well as research problems.

Taking into account these features, the author has developed and implemented an original method to compute the stresses in the cross section of a beam.

2. Theoretical background

Stresses are computed in a given context, taking into account the loads as well as the shape of the cross section. Moreover, the most relevant stresses are in some points which may be located with respect to a given shape of the section. It results that the shape of the section is important and all the problems are related to it. This is why all the algorithmic solutions must take into account the shape of the cross-section.

An initial idea was to consider a set of common use sections whose dimensions are parameterized, figure 1.

As it can be noticed, the solution is not general and the user of a software application must choose a pre-defined shape.

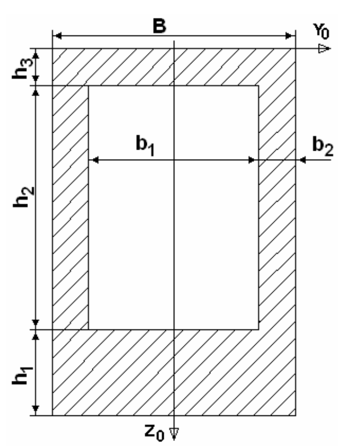

Fig. 1. Parameterized cross-section.

The most general approach is to allow the user to create his own cross-section. As it can be noticed in Fig. 2, a cross-section may be considered as a set of solid and hollow simple shapes. A sign is assigned to each simple shape: $\text{sgn} = -1$ for hollow shapes and $\text{sgn} = +1$ for solid shapes.

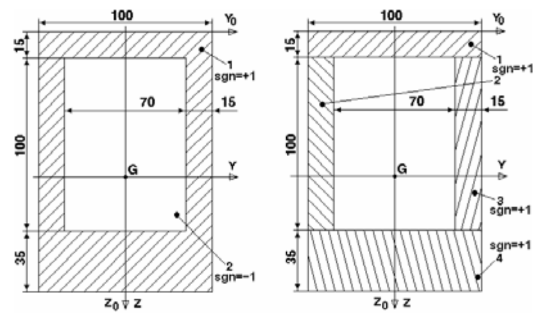

Fig. 2. Two methods to decompose the cross-section.

All the geometrical characteristics must take into account the sign which is initially assigned with respect to the state of the simple shape. The algorithm is:

Stage 1 – the cross section is divided in simple shapes, solid ($\text{sgn} = +1$) or hollow ($\text{sgn} = -1$).

Stage 2 – it is computed the location of the elastic center 'E', and of the center of gravity 'G':

$$\begin{cases} Y_G = \dfrac{\sum_{j=1}^{NB} \text{sgn}_j \cdot \rho_j \cdot A_j \cdot Y_{G\,j}}{\sum_{j=1}^{NB} \text{sgn}_j \cdot \rho_j \cdot A_j} \\ Z_G = \dfrac{\sum_{j=1}^{NB} \text{sgn}_j \cdot \rho_j \cdot A_j \cdot Z_{G\,j}}{\sum_{j=1}^{NB} \text{sgn}_j \cdot \rho_j \cdot A_j} \end{cases} \quad \begin{cases} Y_G = \dfrac{\sum_{j=1}^{NB} \text{sgn}_j \cdot E_j \cdot A_j \cdot Y_{G\,j}}{\sum_{j=1}^{NB} \text{sgn}_j \cdot \rho_j \cdot A_j} \\ Z_G = \dfrac{\sum_{j=1}^{NB} \text{sgn}_j \cdot E_j \cdot A_j \cdot Z_{G\,j}}{\sum_{j=1}^{NB} \text{sgn}_j \cdot \rho_j \cdot A_j} \end{cases} \quad (1),$$

NB is the total number of bodies with simple shape, hollow or solid, from which the section is created. The ρ_j and E_j symbols designate the density, respectively the modulus of elasticity (Young's modulus) of the j-th geometrical body. For homogeneous sections $E \equiv G$.

Stage 3 - the calculus of the distances between the elastic centre of the cross section to the centers of gravity of each body having a simple shape, e_Z, e_Y:

$$\begin{cases} e_{Z\,j} = Y_{G\,j} - Y_E \\ e_{Y\,j} = Z_{G\,j} - Z_E \end{cases} \quad (2).$$

Stage 4 – the calculus of the second moments of inertia or the second moments of area may be done with the formulae:

$$\begin{cases} I_Z = \sum_{j=1}^{NB} I_{Z\,j} \\ I_Y = \sum_{j=1}^{NB} I_{Y\,j} \end{cases}, \quad \begin{cases} I_Z = \sum_{j=1}^{NB} \left(I_{Z\,j}^{local} + e_{Z\,j}^2 \cdot A_j \right) \\ I_Y = \sum_{j=1}^{NB} \left(I_{Y\,j}^{local} + e_{Y\,j}^2 \cdot A_j \right) \end{cases} \quad (3).$$

Stage 5 – calculus of the product moment of area

$$I_{ZY} = \sum_{j=1}^{NB} I_{ZY\,j}, \quad I_{ZY} = \sum_{j=1}^{NB} \left(I_{ZY\,j}^{local} + e_{Z\,j} \cdot e_{Y\,j} \cdot A_j \right) \quad (4)$$

Stage 5 – calculus of the rigidity modulus for the tensile load,

$$S = \sum_{j=1}^{NB} \operatorname{sgn}_j \cdot E_j \cdot A_j \tag{5}$$

Stage 6 – calculus of the rigidity moduli for the bending load

$$\begin{cases} R_Y = \sum_{j=1}^{NB} \operatorname{sgn}_j \cdot E_j \cdot I_{Y\,j} \\ R_Z = \sum_{j=1}^{NB} \operatorname{sgn}_j \cdot E_j \cdot I_{Z\,j} \end{cases} \tag{6}$$

Stage 7 – calculus of the 'centrifugal' rigidity modulus

$$R_{ZY} = \sum_{j=1}^{NB} \operatorname{sgn}_j \cdot E_j \cdot I_{ZY\,j} \tag{7}$$

Once these values are known, the computation of the stresses is the next stage of the algorithm.

In order to have an overview regarding the stresses in the cross-section, there must be used some formulae that take into consideration both stresses, normal and tangential.

Thus, in the point located inside the cross-section, having the (y_j, z_j) coordinates, the stresses may be computed using the expressions:

$$\begin{cases} \sigma_{(y_j,z_j)} = \dfrac{N \cdot E_j}{S} + \dfrac{M_Y \cdot E_j \cdot (R_Z \cdot z_j - R_{ZY} \cdot y_j)}{R_Z \cdot R_Y - R_{ZY}^2} - \dfrac{M_Z \cdot E_j \cdot (R_Y \cdot y_j - R_{ZY} \cdot z_j)}{R_Z \cdot R_Y - R_{ZY}^2} \\ \tau^{T_Z}_{(z)} = \dfrac{T_Z}{I_Y} \cdot \dfrac{S_{Y(z)}}{b_{Y(z)}}; \quad \tau^{T_Y}_{(y)} = \dfrac{T_Y}{I_Z} \cdot \dfrac{S_{Z(y)}}{b_{Z(y)}} \end{cases} \tag{8}$$

The first relation is a generalization of Navier's formula and the tangential stresses are computed by the use of Juravschi's formula. The values in the previous expressions are:

N - axial force;

T_Y, T_Z - shear forces;

M_Y, M_Z - bending moments;

(y_j, z_j) - coordinates of the point belonging to the ' j ' material;

E_j - modulus of elasticity of the ' j ' material;

$S_{Y(z_j)}$, $S_{Z(y_j)}$ current first moment of area;

$b_{Y(z_j)}$, $b_{Z(y_j)}$ current width of the cross section

One can notice that Navier's formula can be used by replacing the coordinates of the point which must be located within the domain. Moreover, one can see that the normal stress has a linear law of variation.

Juravski's formula is more complex and it consists of a constant factor, the ratio between the shear force and the second moment of inertia and a variable one which is the ratio between the first moment of the area and the width of the section. The current first moments $S_{Y(z_j)}$, $S_{Z(y_j)}$, and the current widths of the section $b_{Y(z_j)}$, $b_{Z(y_j)}$, depend on the location of the point within the section where the tangential stress must be computed.

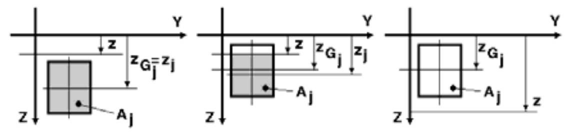

Fig. 3. Three possible cases for the computation of the first moment of area $S_{Y\,j(z)}$ for a simple shape (rectangle).

These values may be computed similar to (1)-(7), by taking into account the solid and the hollow attributes of the simple shape. The relations are:

$$S_{Y(Z)} = \sum_{j=1}^{NB} \operatorname{sgn}_j \cdot S_{Y\,j(Z)} \tag{9}$$

$$b_{Y(Z)} = \sum_{j=1}^{NB} \operatorname{sgn}_j \cdot b_{Y\,j(Z)} \tag{10}$$

Fig. 3 presents three cases considered for the computation of first moment of area for a rectangle simple shape.

The case presented on the leftmost side considers that the whole simple shape is taken into consideration for the computation of the first moment of area. If h_j and b_j are the sides of the ' j ' body, the area is $A_j = h_j \cdot b_j$ and the distance from the center of gravity of the area to the Y axis is $Z_{G\,j}$. The first moment of area for $Z < Z_{G\,j} - \frac{h_j}{2}$ is $S_{Y(Z)} = h_j \cdot b_j \cdot Z_{G\,j}$.

The case presented in the center of figure 3 considers that the current Z coordinate crosses the rectangle. In this case, the sides of the rectangle which is taken into consideration for the calculus of the first moment of area are $h = Z_{G\,j} + \frac{h_j}{2} - Z$ and $b = b_j$. The distance from the center of gravity to the Y axis is

$$Z_j = Z + \frac{Z_{G\,j} + \frac{h_j}{2} - Z}{2} = \frac{Z_{G\,j} + \frac{h_j}{2} + Z}{2} = \frac{1}{2} \cdot \left(Z_{G\,j} + \frac{h_j}{2} + Z \right)$$

The first moment of area for $Z_{G\,j} - \frac{h_j}{2} \le Z \le Z_{G\,j} + \frac{h_j}{2}$ is

$$S_{Y(Z)} = \frac{1}{2} \cdot \left(Z_{G\,j} + \frac{h_j}{2} - Z \right) \cdot \left(Z_{G\,j} + \frac{h_j}{2} + Z \right) \cdot b_j .$$

The case presented in the rightmost side of Fig. 3 considers that the simple shape is not belonging to the region where the first moment is computed. In this case, for $Z > Z_{G\,j} + \frac{h_j}{2}$, the first moment of area is $S_{Y(Z)} = 0$.

Similar formulae can be conceived for several bodies having a 'simple' shape.

2.1 Location of the relevant stresses

A computer approach allows the analyst to consider a large number of points where the stresses are computed in order to create an accurate diagram of the stresses.

When a computer is used to solve such problems it is important to respect the basic rules of the classic discipline: to have relevant and accurate results using a small amount of time. Because time was important and it was not possible to use a computer at that time, there were used simplifying hypotheses and simple methods to calculate the stresses, strains or deflections. At present the computer methods solve the problem regarding the speed and accuracy, but it still must be respected the rule regarding the relevancy of the results. This means that the algorithm must be smart enough to decide what results have a special significance for the phenomenon which is investigated.

In our applied elasticity problem, it can be noticed that the largest values of the stresses are either in the extreme positions with respect to the axes, or in the adjacent area of the center of gravity. Because the cross section is considered a set of bodies, hollow or solid, having simple shapes, the basic idea is to assign some points to the simple shapes, points where the stresses should be computed.

It is important to conceive a set of general rules to identify these points in a cross-section having a complex shape. Also, the rules must be readily used in an algorithm and, further on, implemented in a program. In order to fulfill these requirements, the location of these points is considered on the basis of three principal criteria:

1. geometrical shape of the current body (see Fig. 4, relevant for rectangle shaped ‘simple’ bodies);
2. values of the loads in the current section (see Fig. 5);
3. minimum distance between the points located on the boundary and the adjacent points
4. elastic centre of the cross-section.

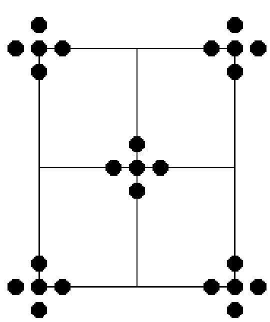

Fig. 4. Location of the relevant points, taking into account the shape of the 'simple' body.

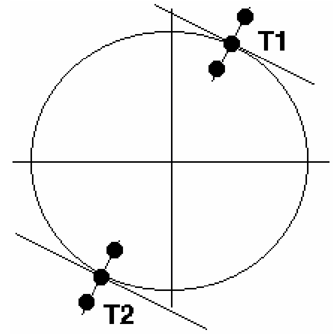

Fig. 5. Location of the relevant points, taking into account the mechanical criteria (slope of the neutral axis).

The first condition is relevant for the bodies which have ‘corners’ that can become the most distant points with respect to the elastic centre or to the neutral axis. The centre of gravity of this type of body is another relevant point because it may become the centre of gravity of the whole section in some certain cases like: for a homogeneous cross-section or for a single geometrical body or for a cross section with two axis of symmetry. Some other relevant points for circle-shaped sections may be found in the intersection point between the axis and the boundary of this body.

The second criterion is relevant for circle-shaped bodies. As an example let us consider the case presented in Fig. 5. The two tangents are parallel with the neutral axis. The points designated with T1 and T2 are important for the diagram of the normal stresses. The slope of the neutral axis is necessary for the tangents as well as for the calculus of the slope of the perpendicular direction. This direction is important because it represents the slope of the ‘zero-reference’ line of the diagram of the normal stresses. Moreover, the slope of the neutral axis is the direction of the lines of the hatch to be used for the diagram of the normal stresses.

The third criterion is important because for each relevant point considered on the basis of the two previous criteria there must be considered some adjacent points in order to express the sudden variation of the tangential stress, if the width of the cross-section has a sudden variation. This is why there is defined a parameter named Eps. The adjacent points are located at Eps distance away from the locations of the relevant points, with respect to the axis. The value of the Eps parameter is very important. It must be large enough to be relevant, this means to generate distinct points and also it must not be considered to be null by the computer. On the other hand, the value must be small enough to assure a proper accuracy of the stresses, especially in design problems where the coordinates are multiplied with the parameter to be computed and which may have a large value.

The fourth criterion is relevant for the cases when the elastic centre is not located in the internal region of the cross-section. As an example, this is the case of the cross-section of a pipe.

3. Results

Based on the previous interdisciplinary approach, was conceived a computer code consisting of more than 12000 lines. The program is parameterized and it creates SCR files used in AutoCAD to automatically draw the cross sections, Fig. 6.

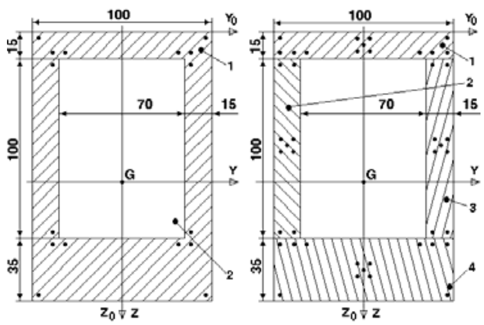

Fig. 6. Different sets of points where the stresses are computed with respect to the methods employed to divide the cross-section in simple shapes.

The results were checked in detail and they were accurate.

The data input is done by the use of a friendly user interface.

The user is allowed to select the units and the materials, from a library and to modify some of the parameters.

4. Discussion

One of the most important problems was to create the list of points where the stresses must be computed. The algorithm consists of several stages:

• all the points of the simple bodies are stored in a file which includes: coordinates, index of the body (shape, geometrical data, material);

• points located outside the cross section are eliminated;

• points must be distinct as coordinates, sgn and as materials;

• for heterogeneous cross-sections it must be decided the material employed for calculus of the normal stress (materials with both $\mathrm{sgn} = +1$ and $\mathrm{sgn} = -1$ are not considered); at the end of this stage all the points are distinct in terms of coordinates and each of them is assigned to an unique material;

• if the elastic centre (gravity center) is not located into the internal area of the section (pipes, concave shapes, multiconnected sections (with holes)), this point is added because it is useful for the calculus of the tangential stresses.

There must be reminded that the user is allowed to set a parameter for each point in order to select the stresses to be computed. For the case previously presented regarding the elastic centre to be added to the final list, in this point may be computed only the tangential stresses.

The generation of the final list of points is a method to filter the quantitative information related to the values of the stresses.

This automatic filtering algorithm might include in the final list of points some points without a great degree of relevancy, but they are only a few.

Taking into account that the cross section is a set of hollow or solid ‘simple’ geometrical bodies, the method to divide the cross-section is important because it can generate less so called 'relevant' points, case presented in Fig. 6.

5. Conclusions

Using the program to solve a large amount of problems, experience shows that the original method is very effective, being flexible and reliable. All the appropriate stages in the algorithm can be conceived using only common-sense conditions.

Handling in an effective way the problem of the units there may be used parameters which allow the use of this method in design problems.

The database which stores the geometry and the materials may be used in alternate studies which employ FEM or FDM, in this way the analyst being allowed to create hybrid models.

This computer based solution can be easily generalised.

Special cases when torsion is one of the loads require particular solutions.

Acknowledgements

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*Corresponding author: eoanta@yahoo.com