

# An ambiguity function aided target density function for radar imaging

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This paper investigates a new approach to target density functions for active sensor imaging. The data obtained from a phased array radar system is processed by the newly developed technique. In order to reconstruct the target density function, the ambiguity function -that is widely used as a radar performance tool- is employed. Theoretical background of the system model is examined and simulation results are achieved.

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## 1. Introduction

Radars are devices that interpret the relation between sent and backscattered signals and receive various information about the target. The distribution of the target's reflectivity function on spatial plane is able to provide the target's radar image [1,2,14]. The range resolution of target area is related to bandwidth, while the cross range resolution varies with the aperture size of radar. However, using large apertures in order to obtain high cross range resolutions is difficult to construct and also expensive. In 1950's a revolutionary invention called *synthetic aperture* radar was made by Wiley [11,12]. This invention provided new possibilities to coherently process the signals obtained from multiple radar elements from diverse angles related to target, or implement various geometrical approaches.

In this paper, system models that fit in with this approach are investigated and a new method is developed.

## 2. Classical approaches

In this paper, two previous studies on target density functions are presented.

### SAR – ISAR Imaging

SAR (Synthetic Aperture Radar) – ISAR (Inverse Synthetic Aperture Radar) systems are powerful signal processing techniques with a wide range of applications that adapted various forms of problems. The basic goal of SAR-ISAR systems is to achieve a two dimensional planar image of the observed target. Synthetic aperture methods are used in order to obtain raw signals in SAR-ISAR systems.

The densities of scattering centers can be shown on a two-dimensional planar image by applying Fourier based operations to the signals obtained under finite bandwidth and diverse aspect [1,4,9,11,14,15].

In this method, the basic steps of the ISAR approach that processes a three dimensional target's two dimensional image can be considered as:

$$s(t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(x, y) e^{-j2\pi f_0 \frac{2R_p(t)}{c}} dx dy \quad (1)$$

This equation assumes the condition that  $2R_p(t)/c \leq T \leq T_{PRI} + 2R_p(t) \leq T f_0$ . The term  $\rho(x, y)$  is the density function of the target being imaged,  $T_{PRI}$  is pulse repetition interval,  $R_p(t)$  is range,  $f_0$  is carrier frequency and the  $c$  is the speed of propagation, (i.e., light speed). Range can be expressed as a function of time.

$$R_p(t) = R(t) + x \cos[\theta(t) - \alpha] - y \sin[\theta(t) - \alpha] \quad (2)$$

In this expression  $\alpha$  is azimuth,  $\theta(t)$  is the rotation angle as a function of time. After these operations, by taking inverse Fourier transform of  $s(t)$ , the  $\rho(x, y)$  target density functions can be reconstructed as [4].

$$\rho(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} X(f_x, f_y) e^{j2\pi(x f_x - y f_y) f_0 \frac{2R_p(t)}{c}} df_x df_y \quad (3)$$

$$f_x = \frac{2f_0}{c} \cos \theta(t) \text{ and } f_y = \frac{2f_0}{c} \sin \theta(t) \quad (4)$$

### Fowle – Naparst Approach

The Fowle-Naparst's work is one the first ideas that put target density function forward [7]. In accordance with this work, the density of the space which includes target is

reconstructed via the ambiguity function. The expression of the ambiguity function is given as;

$$A(x, y) = \int_{-\infty}^{\infty} u\left(t - \frac{x}{2}\right) \bar{v}\left(t + \frac{x}{2}\right) e^{-j2\pi y t} dt \quad (5)$$

In this function,  $x$  and  $y$  are the range and velocity of the target respectively. This approach was first introduced by Fowle [6] and then improved by Naparst [7] by employing the ambiguity function. In this work, an improved target density function  $D(x,y)$  employing the ambiguity function which includes the variables velocity and range was developed.

The relation between the signal reflected from the target  $e(t)$  and the signal sent  $s(t)$  is expressed as

$$e(t) = \int_0^{\infty} \int_{-\infty}^{\infty} D(x, y) \sqrt{y} s(y(t-x)) dx dy \quad (6)$$

The target density function  $D(x,y)$  that contains by the backscattered signal can be reconstructed by the following complex operations in vector space including the ambiguity function given above.

$$D(x, y) = \sum_{n,m=0}^{\infty} \langle e_n, s_m \rangle A_{nm}(x, y) \quad (7)$$

In this expression  $s_m$  is the propagated signal and  $e_n$  is the backscattered signal (echoed). The ambiguity function used has the following form.

$$A_{nm}(x, y) = \int_{-\infty}^{\infty} s_n(y(t-x)) \bar{s}_m(t) dt \quad (8)$$

Fig. 1 represents an approximation of a Dirac function's target density function, for  $x=0$   $y=1$ , that is reconstructed by ambiguity function based Fowle-Naparst approach [7].

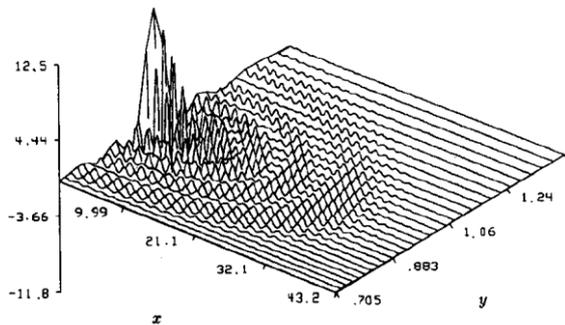


Fig. 1.  $D(x,y)$  approximation for  $x=1, y=0$ .

### Imaging by range-angle target density function

In this paper, a new target density function that has different properties from the target density functions

discussed above is presented.  $G(R,\beta)$  formed new target density function contains the variables  $R$  as range and  $\beta=\cos(\theta)$  as function of scanning angle. The newly defined target density function  $G(R,\beta)$  represents the ratio of the amplitudes of the signal sent towards the point  $(R,\beta)$  to the backscattered signal.

The new approach introduced in this paper is imaging with phased array radar system. In this case our imaging scenario's schematic representation becomes:

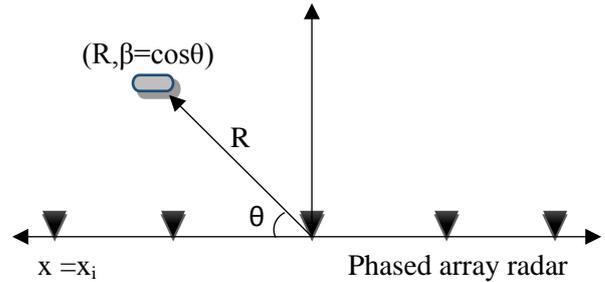


Fig. 2. Imaging with linear phased array radar.

As can be seen, this approach only takes the upper half plane of the coordinate system into account [3]. In this condition, the propagated pulse signal  $p(t)$  is:

$$p(t) = \sum_{k=-\infty}^{\infty} A_k e^{j k \omega_0 t} \quad (9)$$

$$\omega_0 = 2\pi \times PRF \quad (10)$$

PRF term in this equation is the *pulse repetition frequency*. If this signal is modulated with  $s_c(t)$  carrier signal which is expressed as:

$$s_c(t) = e^{j \omega_c t} \quad (11)$$

the modulated output signal becomes:

$$s_m(t) = p(t) s_c(t) \quad (12)$$

Echo from the point represented by  $G(R,\beta)$  density function is:

$$s_r(x, t) = s_m(t - 2R/c - \beta x/c) g(R, \beta) \quad (13)$$

In this case, if the backscattered signal is the echo of the point scatterer located at  $R_1$ , the signal that contains the image function can be showed as;

$$\begin{aligned}
s_r(x, t) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s_m(t - 2R/c - \beta x/c) g(R, \beta) dR d\beta \\
&= \int_{-1}^1 \int_0^{R_1} s_m(t - 2R/c - \beta x/c) g(R, \beta) dR d\beta \\
&= \int_{-1}^1 \int_0^{R_1} p(t - 2R/c - \beta x/c) \\
&\quad \times e^{j\omega_c(t - 2R/c - \beta x/c)} g(R, \beta) dR d\beta \\
&= \int_{-1}^1 \int_0^{R_1} p(t - 2R/c - \beta x/c) \\
&\quad \times e^{-j\omega_c(2R/c + \beta x/c)} e^{j\omega_c t} g(R, \beta) dR d\beta
\end{aligned} \tag{14}$$

The term  $s(x, t)$  is the output of the radar system which consists of phased array elements. For the solution to the algorithm, the next steps can be evaluated.

$$\begin{aligned}
s_r(x, t) &= \int_{-1}^1 \int_0^{R_1} \sum_{k=-\infty}^{\infty} A_k e^{j k \omega_0 (t - 2R/c - \beta x/c)} \\
&\quad \times e^{-j\omega_c(2R/c + \beta x/c)} e^{j\omega_c t} g(R, \beta) dR d\beta \\
&= \sum_{k=-\infty}^{\infty} A_k e^{j k \omega_0 t} \int_{-1}^1 \int_0^{R_1} e^{-j k \omega_0 (2R/c + \beta x/c)} \\
&\quad \times e^{-j\omega_c(2R/c + \beta x/c)} e^{j\omega_c t} g(R, \beta) dR d\beta \\
&= \sum_{k=-\infty}^{\infty} A_k e^{j(\omega_c + k\omega_0)t} \int_{-1}^1 \int_0^{R_1} e^{-j k \omega_0 (2R/c + \beta x/c)} \\
&\quad \times e^{-j\omega_c(2R/c + \beta x/c)} g(R, \beta) dR d\beta \\
&= \sum_{k=-\infty}^{\infty} A_k e^{j(\omega_c + k\omega_0)t} \int_{-1}^1 \int_0^{R_1} e^{-j(\omega_c + k\omega_0)(2R/c + \beta x/c)} \\
&\quad \times g(R, \beta) dR d\beta
\end{aligned} \tag{15}$$

If the last equation that expresses the radar output is demodulated by  $s_d(t)$ ;

$$\begin{aligned}
s_d(t) &= e^{-j(\omega_c + k\omega_0)t} / A_k \\
s_r(x, t) &= (e^{-j(\omega_c + k\omega_0)t} / A_k) \left[ \sum_{k=-\infty}^{\infty} A_k e^{j(\omega_c + k\omega_0)t} \right. \\
&\quad \left. \times \int_{-1}^1 \int_0^{R_1} e^{-j(\omega_c + k\omega_0)(2R/c + \beta x/c)} g(R, \beta) dR d\beta \right] \\
S(k, x) &= \int_{-1}^1 \int_0^{R_1} e^{-j(\omega_c + k\omega_0)(2R/c + \beta x/c)} g(R, \beta) dR d\beta \\
&= \int_{-1}^1 \int_0^{R_1} e^{-j(\omega_c + k\omega_0)(2R/c + \beta x/c)} g(R, \beta) dR d\beta
\end{aligned} \tag{16} \tag{17}$$

Last equation can be considered again as  $G(k, \beta)$ , for the  $k$  and  $\beta$  variables;

$$G(k, \beta) = \int_0^{R_1} g(R, \beta) e^{-j(\omega_c + k\omega_0)2R/c} dR \tag{18}$$

$$S(k, x) = \int_{-1}^1 G(k, \beta) e^{-j(\omega_c + k\omega_0)\beta x/c} d\beta \tag{19}$$

$$S_k(x) = \int_{-1}^1 G_k(\beta) e^{-j(\omega_c + k\omega_0)\beta x/c} d\beta \tag{20}$$

The important matter in here is to extract the target density function  $G(R, \beta)$ . Therefore equation (20) can be considered as;

$$G_k(\beta) = \int_0^{R_1} g(R, \beta) e^{-j(\omega_c + k\omega_0)2R/c} dR \tag{21}$$

To obtain the target density function from equation (21), the ambiguity function discussed in previous chapters (equation (5)) is employed. By considering the signals  $s_m(t)$  and  $s_n(t)$ , we can make use of the correlation – power spectral density relation.

The correlation between  $s_m(t)$  and  $s_n(t)$  signals can be expressed as:

$$R(\tau) = s_m(t) * s_n(-t) = \int_{-\infty}^{\infty} s_m(\tau) \bar{s}_n(t + \tau) dt \tag{22}$$

and in the symmetrical form;

$$R(\tau) = \int_{-\infty}^{\infty} s_m\left(\tau - \frac{t}{2}\right) \bar{s}_n\left(\tau + \frac{t}{2}\right) dt. \tag{23}$$

The power spectral density corresponding to the target is correlation function's Fourier transform, then the next steps can be formed:

$$\begin{aligned}
S(\omega) &= \int_{-\infty}^{\infty} R_x(\tau) e^{-j\omega\tau} d\tau \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(\tau) \bar{v}(t + \tau) dt e^{-j\omega\tau} d\tau \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(\tau) \bar{v}(t + \tau) e^{-j\omega\tau} d\tau dt
\end{aligned} \tag{24}$$

for  $\tau = \tau - \frac{t}{2}$  ;

$$\begin{aligned}
S(\omega) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u\left(\tau - \frac{t}{2}\right) \bar{v}\left(\tau - \frac{t}{2} + t\right) e^{-j\omega\left(\tau - \frac{t}{2}\right)} d\tau dt \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u\left(\tau - \frac{t}{2}\right) \bar{v}\left(\tau + \frac{t}{2}\right) e^{-j\omega\tau} d\tau e^{j\omega\frac{t}{2}} dt \\
&= \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} u\left(\tau - \frac{t}{2}\right) \bar{v}\left(\tau + \frac{t}{2}\right) e^{-j\omega\tau} d\tau \right) e^{j\omega\frac{t}{2}} dt
\end{aligned} \tag{25}$$

$$A(t, \omega) = \int_{-\infty}^{\infty} u\left(\tau - \frac{t}{2}\right) \bar{v}\left(\tau + \frac{t}{2}\right) e^{-j\omega\tau} d\tau \tag{26}$$

$$S(\omega) = \int_{-\infty}^{\infty} A(t, \omega) e^{j\omega\frac{t}{2}} dt \tag{27}$$

Equation (21) that contains the  $G(R, \beta)$  target density function is compared with the equation (27);

$$g(R, \beta) \equiv A(t, \omega) \tag{28}$$

Therefore the target density function can be expressed as:

$$G(R, \beta) = \int_{-\infty}^{\infty} s_m \left( t - \frac{\tau_R}{2} \right) \overline{s_n \left( t + \frac{\tau_R}{2} \right)} e^{j\beta t} dt \quad (29)$$

for  $\tau = \frac{R}{2c}$ .

As a result, target density function is obtained by analyzing the relationship between ambiguity function and Correlation (Power Spectral Density) approaches.

### 3. Simulation

The simulated target is a fighter aircraft, digitally constructed by 120 point scatterers. The dataset of the simulated signal represents a complex matrix of the signal received by each unit of phased array system. And each received data has 64 samples [15].

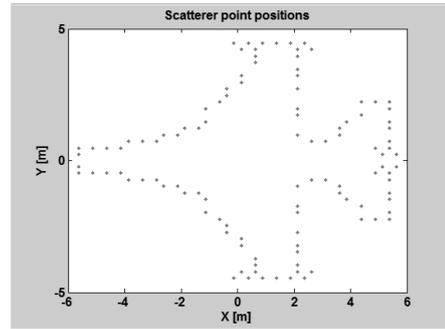


Fig. 3. Positions of the scatterer centers.

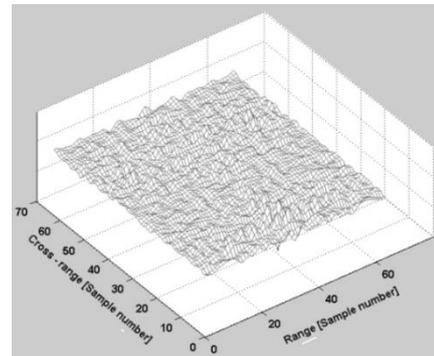


Fig. 4. Backscattered E-field from scattering centers.

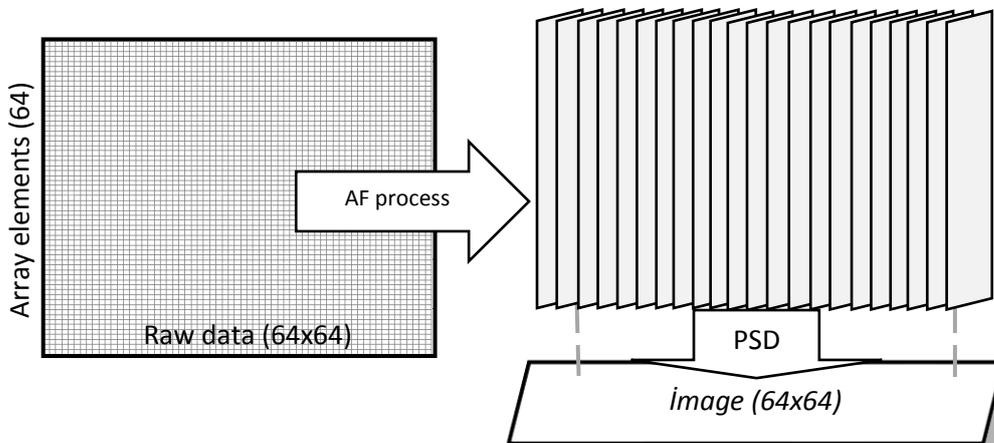


Fig. 5. Schematic representation of imaging process.

The radar operates at 8 GHz frequency with 525 MHz bandwidth. The total aspect angle diversity is 13.5°.

In Fig. 5, schematic presentation of the process is given.

During the ambiguity function process, only the right half plane of the ambiguity function is taken due to the symmetrical property of the ambiguity function.

Distribution and intensity of the side lobes around the peak points in the reconstructed image are directly related with signal choice [5,13].

In the imaging process of the ambiguity function, we expect a sharp peak to be on the scatterer point. However, this is impossible due to the natural imperfections of theoretical and computational processes such as finite bandwidth and noise [16].

#### 4. Result

Fig. 6 shows the result image of reconstruction of the target density function by ambiguity function. In Fig. 7, ISAR image of the target is also given for comparison.

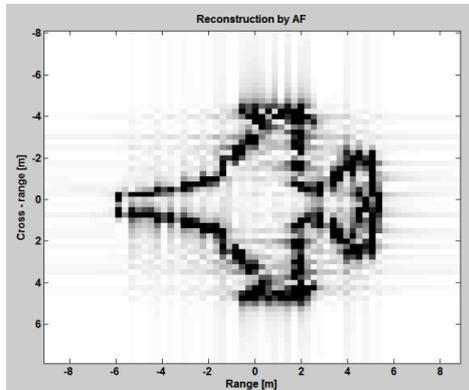


Fig. 6. Image of target density function reconstructed by ambiguity function.

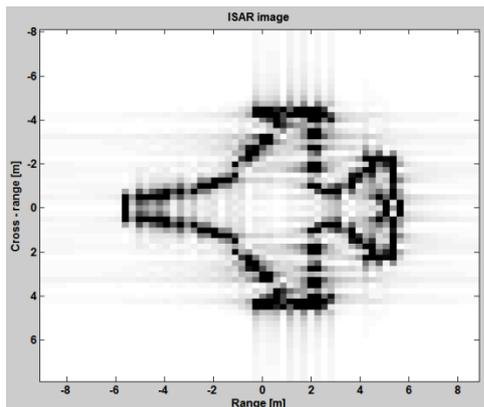


Fig. 7. Conventional ISAR image.

Alterations caused by the side lobes can be seen clearly in Fig. 8 and Fig. 9.

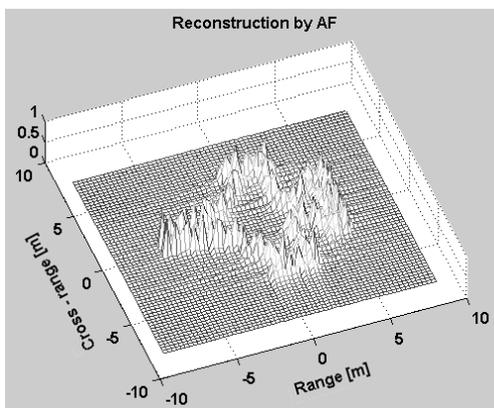


Fig. 8. Normalized scatterer points intensity of target density function that reconstructed by ambiguity function.

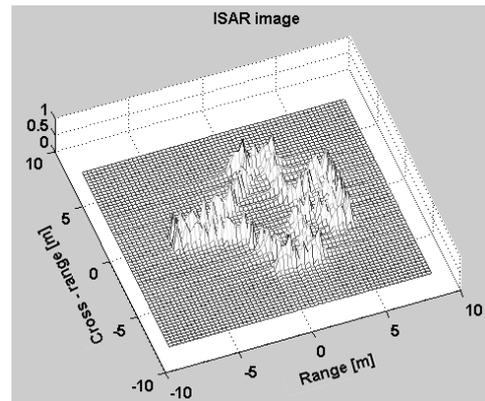


Fig. 9. ISAR image.

This technique certainly supports fundamental signal processing methods such as denoising, interpolation etc. Improvements on the reference signal enables us to get a finer result [13,16].

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