A unifying approach to track-to-track correlation for multisensor fusion for multiple targets

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In this paper, a global modeling approach was proposed for multi sensor fusion problems. Once the global model was investigated considering the data association and fusion, it is adapted to track to track correlation problem by a new approach. The key development of the approach is that a decentralized filtering algorithm is used for data fusion and state estimation problems in a multi-target tracking system. The use of a global mapping matrix for the track to track correlation is key element of our technique. Via the presented mathematical models, the sensor fusion and track to track correlation problems can be solved in a global way.

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1. Introduction

The multi-target tracking systems using measurement data from multiple sensors have been investigated in a number of papers [1-9]. The major purpose of a multi-sensor fusion approach is to complement the data of one sensor with that of another sensor in order to obtain better target measurement information and to make a more accurate estimation.

Many advantages are obtained from the use of multiple sensors in a target surveillance system. A global modeling and a track to track correlation approach for multisensor problems are presented in this paper. They are also applied to multi-target tracking problems. A tracking system that includes more varied data from multiple sensors will greatly improve data association.

The major problem in the fusion of track information is obviously the determination of overall distinct targets by the central tracking processor and for it to let each sensor know which ones among all the targets it is tracking [10-25]. In our approach, a decentralized estimation algorithm with global modeling definitions is investigated. In order to solve the track-to track correlation problems, a mapping matrix is developed which arises in decentralized filtering which indicates the targets tracked by each individual local sensor. Utilizing this approach with local tracking systems, each sensor can perform a local data association and filtering on the group of targets that it is observing. It produces its own estimate of the kinematic quantities associated with these targets. Then it transmits these estimates to a central processor, which combines this information to produce a global estimate of all targets. Via the proposed technique, the data fusion and track to track correlation are thus achieved simultaneously.

2. Preliminaries

Data fusion techniques are used in many tracking and surveillance systems to complement the data of one sensor with that of another sensor in order to obtain better target measurement information and to make a more accurate estimation [26-32]. Multisensor data fusion seeks to combine data from multiple sensors to perform inferences that may not be possible from a single sensor alone.

The general attempts in multi sensor fusion problems is to employ a number of sensors and to fuse the information obtained from all these sensors on a central processor. A general multisensor tracking system is denoted in Fig. 1.

The sensor-target space is in multiple form which includes multiple sensors and targets. In reality, the configuration in Fig. 1a is replaced by the 1b. Each sensor is spaced to see a few of the targets. Thus, some of the view of the fields can be joint. Any sensor can see the targets simultaneously seen by the other sensors at the same time. Detection of the targets seen by more than one sensor is a main consideration of the multisensor problems for multitarget tracking. In this work, a global approach is presented for the multisensor track to track correlation problems.

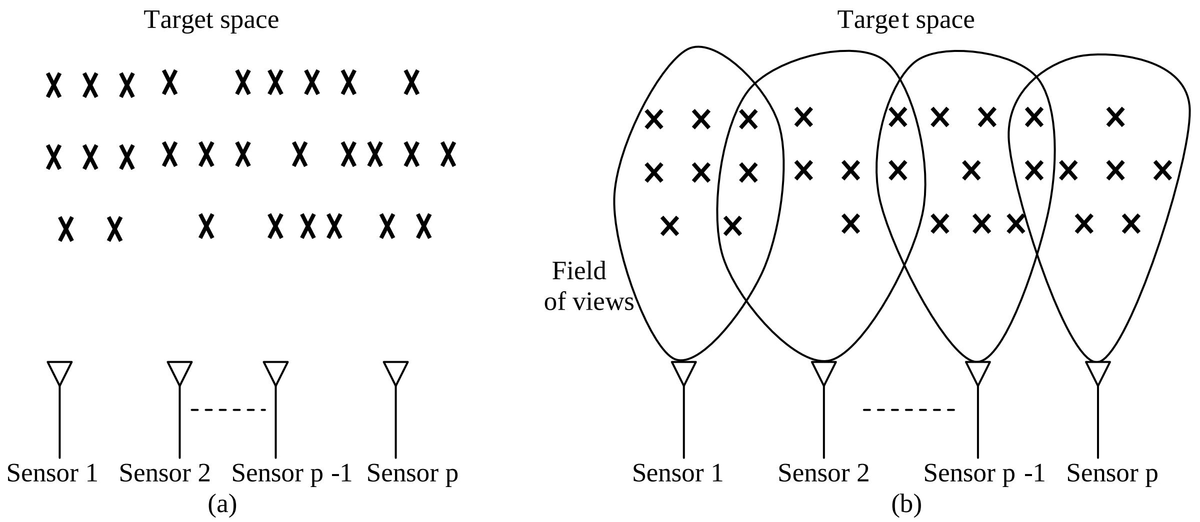

Fig. 1. View of multi sensor - multi target space.

2.1 Centralized and decentralized architectures

The general attempts in multi sensor fusion problems is to employ a number of sensors and to fuse the information obtained from all these sensors on a central processor. General solutions to the multi-sensor fusion problems were in two architectures. These were centralized and decentralized mechanizations respectively considered as the following Fig. 2, [10-17].

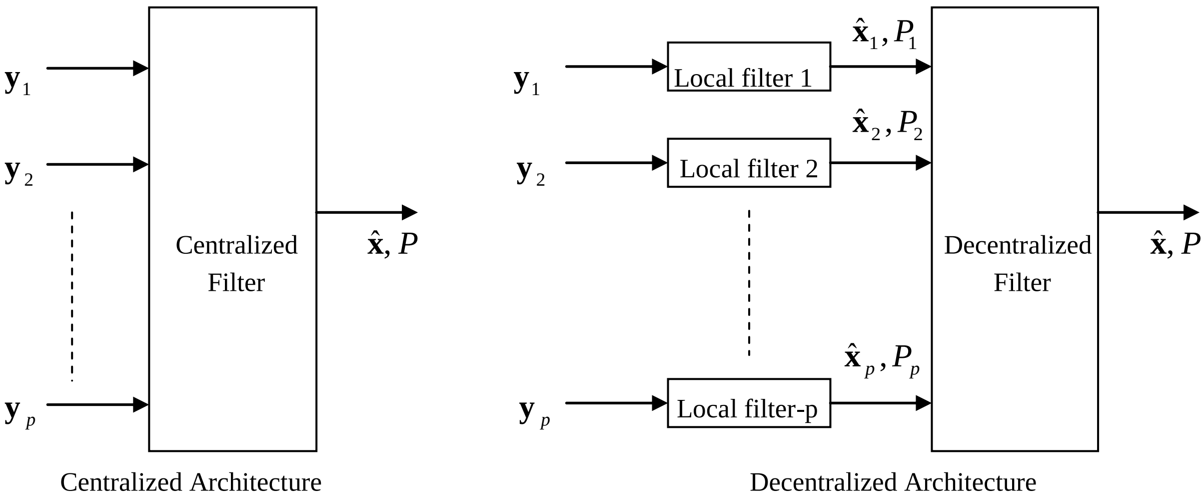

Fig. 2. Centralized and decentralized Filters.

The principles of both are analyzed taking consideration into the general and Kalman filtering approaches. First works in this field were based on the centralized network which is an organization of a feedback from the central processor to local processor units each of which includes a sensor and a local processor. Local estimations are then generated from the global estimation obtained from the previous step. The general configuration of this is given in Fig. 2.

However, because of dense of computations in this configuration decentralized estimation algorithm was developed based on parallelization of the Kalman filter equations. The decentralized Kalman filtering is well-known way in multi-sensor fusion problems that obtains the global estimation using only local estimates without transmission of information between sensors [18-25]. This was advantageous compared to the first attempts. In addition to the decentralized Kalman filter, the federated filter and the Bayesian were also alternative popular multi sensor fusion methods.

All the methods described above require the use of the central processor in order to fuse information obtained by the sensors. The main disadvantage of this approach is that in the case of central processing failure, the overall system will also fail.

The fundamental concept of this approach which we developed for sensor fusion of multiple targets is that track to track correlation is achieved using a global modeling approach, exploiting certain results from decentralized filtering. The central tracking system processes the track estimates formed by local tracking filters rather than the original measurement data. The simple decentralized filtering structure is shown in Fig. 3.

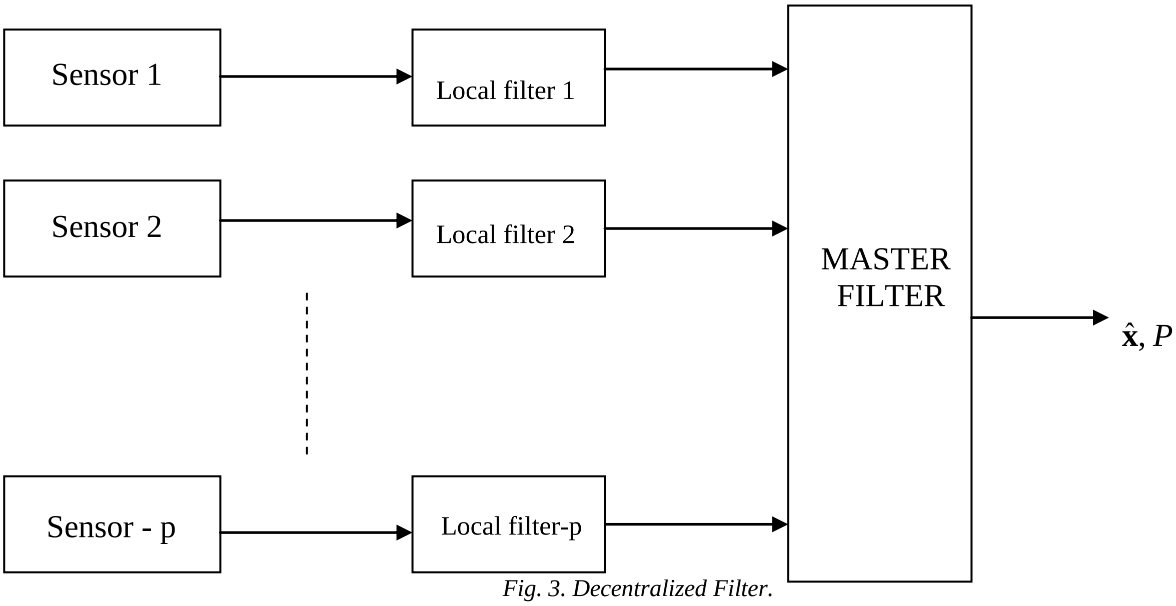

Fig. 3. Decentralized Filter.

The fundamental concept of this approach is that track to track correlation is achieved using a global modeling approach, exploiting certain results from decentralized filtering. The central tracking system processes the track estimates formed by local tracking filters rather than the original measurement data. The general decentralized filtering structure is shown in Fig. 3.

The formulation of such a situation based on a global modeling approach whose details are given below. Suppose that there are p distributed sensors in a tracking system, total n targets in the surveillance region, and n targets seen by the i-th sensor. The global target dynamic model and measurement model of the central tracking system are defined as [18-25].

$$\begin{aligned} X(t+1) &= F(t)X(t) + G(t)W(t) \\ Y_i(t) &= C_i(t)X(t) + V_i(t), \qquad i = 1, \cdots, p \end{aligned} \tag{1}$$

Where,

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_p(t) \end{bmatrix}, \; F(t) = \begin{bmatrix} F_1 & 0 & 0 & 0 \\ 0 & F_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & F_p \end{bmatrix},$$

$$G(t) = \begin{bmatrix} G_1 & 0 & 0 & 0 \\ 0 & G_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & G_p \end{bmatrix}, \; W(t) = \begin{bmatrix} \omega_1(t) \\ \omega_2(t) \\ \vdots \\ \omega_p(t) \end{bmatrix} \tag{2}$$

$$Y_i(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_{p_i}(t) \end{bmatrix}, \; C_i(t) = \begin{bmatrix} c_1(t) \\ c_2(t) \\ \vdots \\ c_{p_i}(t) \end{bmatrix},$$

$$V_i(t) = \begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_{p_i}(t) \end{bmatrix}$$

The target kinematic models of local tracking systems associated with the i-th local sensor are defined as [10-13, 15-25].

$$\begin{aligned} X_i(t+1) &= F_i(t)X_i(t) + G_i(t)W_i(t) \\ Y_i(t) &= C_i(t)X_i(t) + V_i(t), \qquad i = 1, \cdots, p \end{aligned} \tag{3}$$

$X_i(t)$ is the state vector corresponding to the i-th sensor. F_i and G_i are the transition matrix and the noise gain matrix for the i-th sensor, respectively. Y_i is the measurement vector received by the *i*-th sensor. H_i is the measurement matrix corresponding to the *i*-th sensor. W_i and V_i are system noise and measurement noise associated with the targets seen by the i-th sensor, assumed to be normally distributed with the zero mean, and mutually uncorrelated. Moreover, a mapping matrix is defined to associate the local sensors with the central processor, which offers the track to track correlation information. The relationship is defined as

$$C_i(t) = H_i(t)D_i(t) \tag{4}$$

$D_i(t)$ reflects which of the targets are observed by the i-th sensor, and is a quantity to be estimated. In addition, in order to solve the multi-target tracking problem, a data association algorithm denoted 1-step maximum a-posteriori estimate whose details are applied and briefly described here.

2.2 Data Association:1-step maximum a-posteriori estimate

Data association is a general way of finding the tracks of the targets. For each $t = 0,1,2,\cdots$ once a measurement vector is received, the corresponding 1-step a-posteriori probability denoted as a weighting coefficient for each hypothesis can be obtained from one formula derived as follows. Let

$$\begin{aligned} Y^t &= \{Y(0), Y(1), \cdots, Y(t)\}, \\ \beta^t &= \{\beta(0), \beta(1), \cdots, \beta(t)\}, \\ \beta_i^t &= \{\beta_i(0), \beta_i(1), \cdots, \beta_i(t)\} \end{aligned} \tag{5}$$

where β^t is the vector consisting of uncertain parameters and β_i^t is one possible fixed hypothesis. According to the Bayesian rule, the a-posteriori probability of β_i^t conditioned on β_i^{t-1} and Y^t can be computed as follows:

$$p(\beta_i(t) \mid \beta_i^{t-1}, Y^t) = \frac{p(Y(t) \mid \beta_i^{t-1}, Y^{t-1}) p(\beta_i(t) \mid \beta_i^{t-1})}{p(Y(t) \mid \beta_i^{t-1}, Y^{t-1})} = \frac{p(Y(t) \mid \beta_i^{t-1}, Y^{t-1}) p(\beta_i(t) \mid \beta_i^{t-1})}{\sum_k p(Y(t) \mid \beta_i^t) p(\beta_k(t) \mid \beta_i^{t-1})} \tag{6}$$

where the denominator is a constant chosen for each given hypothesis to normalize denoted $1/C$, and thus

$$p(\beta_k(t) \mid \beta_i^{t-1}, Y^t) = C\, p(Y(t) \mid \beta_k(t), \beta_i^{t-1}, Y^{t-1}) p(\beta_k(t) \mid \beta_i^{t-1}) \tag{7}$$

The first term is obtained using the 1-step prediction via the Kalman filter based on each hypothesis, and the second term reflects a priori statistical knowledge of β^t. Thus, the suboptimal Bayesian state estimate can be computed as

$$X(t \mid t) = \sum_k p(\beta_k(t) \mid \beta_i^{t-1}, Y^t) X(t \mid t, \beta_i^{t-1}, \beta_k(t)) \tag{8}$$

The 1-step conditional maximum a-posteriori estimate of $\beta(t)$ is $\beta_k(t)$, so that Equation (8) is maximum.

2.3 Fusion

The fusion concept is known with various names such as data or sensor fusion. Data fusion is the process of combining signals from several sensors into a single world view. Data fusion techniques combine data from multiple sensors, and related information from associated databases, to achieve improved accuracies and more specific inferences than could be achieved by the use of a single sensor alone [26-32]. Fusion is conducted by considering the one of the architectures in Fig. 2 or 3 that each local sensor performs its own data association and estimation for those targets seen by it.

A time-varying matrix $D_i(t)$ used to associate the local processors with the central processor and for track to track correlation is defined in our fusion algorithm. The $D_i(t)$ arises in decentralized filtering which indicates the targets are tracked by each individual local sensor. The fusion algorithm in the central processor is applied to combine the local sensor track results with this matrix. Such a matrix is defined as follows:

$$D_i(t) = [D_i^{jk}], \quad j = 1,2,\cdots,n_i \quad k = 1,2,\cdots,n \tag{9}$$

where M_i^{jk} is an n by n block that indicates which targets are seen by the i-th sensor, and the n is the order of state vector for each target. For example, the jk-th block is set to be the identity matrix if the k-th target is seen by this sensor.

In this approach, the estimates can be computed using an adaptive filter composed of a bank of Kalman filters, which is based on the global model and process all measurements. The summary of fusion algorithm with a decentralized estimation approach for multi – target tracking is shown as follows:

Suppose that there are *p* distributed sensors in a system, total *n* targets in the surveillance region, and n_i targets are seen by the i-th sensor. The discrete—time target dynamic model and measurement model of the tracking system are defined as [18-25].

Global model :

$$X(t+1) = F(t)X(t) + G(t)W(t)$$
$$Y_i(t) = C_i(t)X(t) + V_i(t) \ , \qquad i = 1, \cdots, p \tag{10}$$

Local models :

$$X_i(t+1) = F_i(t)X_i(t) + G_i(t)W_i(t)$$
$$Y_i(t) = H_i(t)X_i(t) + V_i(t) \ , \qquad i = 1, \cdots, p \tag{11}$$

Mapping :

$$C_i(t) = H_i(t)D_i(t) \ , \qquad i = 1, \cdots, p \tag{12}$$

Local Filters

$$P_{i\,f}(t\,|\,t-1) = F_i(t)P_{i\,f}(t-1\,|\,t-1)\,F_i^T(t) + G_i(t)\,Q_i(t)\,G_i^T(t) \tag{13}$$

$$K_{i\,f}(t) = P_{i\,f}(t\,|\,t-1)\,H_i^T(t)\left[H_i(t)\,P_{i\,f}(t\,|\,t-1)H_i^T(t) + R_i(t)\right]^{-1} \tag{14}$$

$$A_{i\,f}(t) = [\,I - K_{i\,f}(t)\,H_i(t)\,]\,F_i(t) \tag{15}$$

$$X_{i\,f}(t\,|\,t) = A_{i\,f}(t)\,X_{i\,f}(t-1\,|\,t-1) + K_{i\,f}(t)Y_i(t) \tag{16}$$

$$P_{i\,f}(t\,|\,t) = [I - K_{i\,f}(t)H_i(t)\,]\,P_{i\,f}(t\,|\,t-1) \tag{17}$$

Central Combining Filter

$$P_f(t\,|\,t-1) = F(t)\,P_f(t-1\,|\,t-1)\,F^T(t) + G(t)\,Q(t)\,G^T(t) \tag{18}$$

$$K_i(t) = P_f(t\,|\,t-1)\,C_i^T(t)\left[C_i(t)\,P_f(t\,|\,t-1)\,C_i^T(t) + R_i(t)\right]^{-1} \tag{19}$$

$$B_f(t) = \left[\sum_{i=1}^{p} D_i^T(t)D_i(t)\right]^{-1} \tag{20}$$

$$P_f(t\,|\,t) = \left[I - \sum_{i=1}^{p} K_i(t)C_i(t)B_f(t)\right]P_f(t\,|\,t-1) \tag{21}$$

$$A_f(t\,|\,t) = \left[I - \sum_{i=1}^{p} K_i(t)C_i(t)B_f(t)\right]F(t) \tag{22}$$

$$\Phi_i(t) = P_f(t\,|\,t)D_i^T(t)P_{i\,f}^{\mathrm{T}}(t\,|\,t) \tag{23}$$

$$T_i(t) = \Phi_i(t)A_{i\,f}(t) \tag{24}$$

$$SI(t\,|\,t) = A_f(t)X_f(t-1\,|\,t-1) - \sum_{i=1}^{p} T_i(t)X_{i\,f}(t-1\,|\,t-1)B_f(t) \tag{25}$$

$$X_f(t\,|\,t) = SI(t\,|\,t) + \sum_{i=1}^{p} \Phi_i(t)X_{i\,f}(t\,|\,t)B_f(t) \tag{26}$$

2.4 Track-to-track vorrelation

In general, the track to track correlation is the way to separate the same targets seen by various sensors [33-43]. Thus, it is the most important part of the multisensor problems. Sometimes it is known with the name of "sensor to sensor correlation" As explained in the previous sections, in the distributed multisensor problems, each local nodes has a data processing system, it processes the information from sensors and transforms the result to data fusion center. The main task of the data fusion center is to process information from different nodes and to judge which of tracks are most probably from the same target.

In the distributed multi-sensor information fusion system, the track-to-track correlation is one of the key techniques and is also the precondition for implementing track fusion, and the correctness of correlation judged would directly impact on the performance of the whole fusion system.

The multisensor track-to-track correlation problem of track-to-track correlation arises when several sensors carry out surveillance over a common volume and each sensor has its own data processing system. In such a scenario each system has a number of tracks and it is necessary to decide whether two tracks from different systems represent the same target.

As for radar problems, in a distributed radar network, the spatially distributed radars send their local tracks to the fusion center. Because every radar provides multi tracks corresponding to multi targets, the entire numbers of local tracks sending to fusion center is much larger than the numbers of real target. In fusion center, the global tracks are created by track fusion based on correlation of these local tracks. In this process, multi-track correlation problem must be simplified to track-to-track correlation problem. The track-to-track correlation problem is proposed in [1-5]. In fusion center, the traditional multi-track correlation method is very time-consuming. High computational load is not feasible in engineering practice. As we need a new method which can reduce the computational load and at the same time can retain the track correlation accuracy, here a track to track correlation method includes the solution to the emphasized problem is investigated by a global approach in the following section.

3. Track-to-track correlation for multisensor fusion for multiple targets

One important problem facing a multiple sensor tracking system in a multiple target environment is the unique identification of targets observed by more than one sensor, if any. The method developed here is to directly correlate the set of measurements from the i-th sensor with those of other possible sensors.

We present a global track-to-track correlation approach by taking advantage of the proposed multisensor fusion techniques in the previous section. The approach is developed by conducting the mapping matrix by a various way. The sensor-target plane of the approach is as the following Fig. 4.

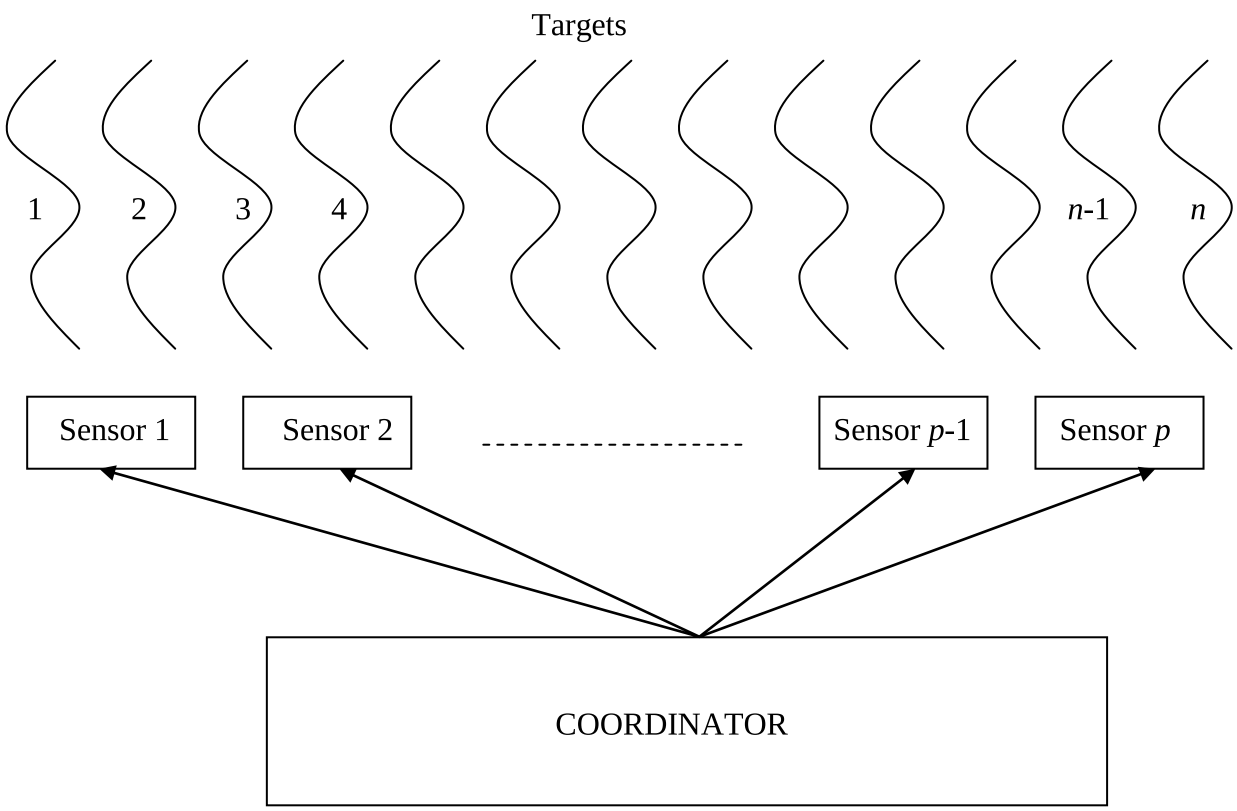

Fig. 4. Global view of multisensor - multitarget space.

The targets in the figure are first assumed distinct and the coordinate system is taken as the cartesian coordinate system of the coordinator. The expression of multisensor fusion for multiple targets is derived from the following general system state-space equations.

$$\begin{aligned} X(t+1) &= F\ X(t) + W(t) \\ Y(t) &= H\ X(t) + V(t) \end{aligned} \tag{27}$$

where $X(t)$ is the state vector of the targets in the spectrum of each sensor, F is the state coefficient, $W(t)$ is the process noise with zero-mean Gaussian, $Y(t)$ is the system output depicts the targets seen by each sensor, H is the coefficient as the transition matrix, and $V(t)$ is the measurement noise with zero-mean Gaussian. As noted, once the assumption of the data association process of the system parameters as $F(t)$ and $H(t)$, they are considered in time invariant forms as F and H.

Here, the proposed global track to track correlation approach based on the Fig. 4 is investigated by considering the decentralized Kalman architecture as the following Fig. 5.

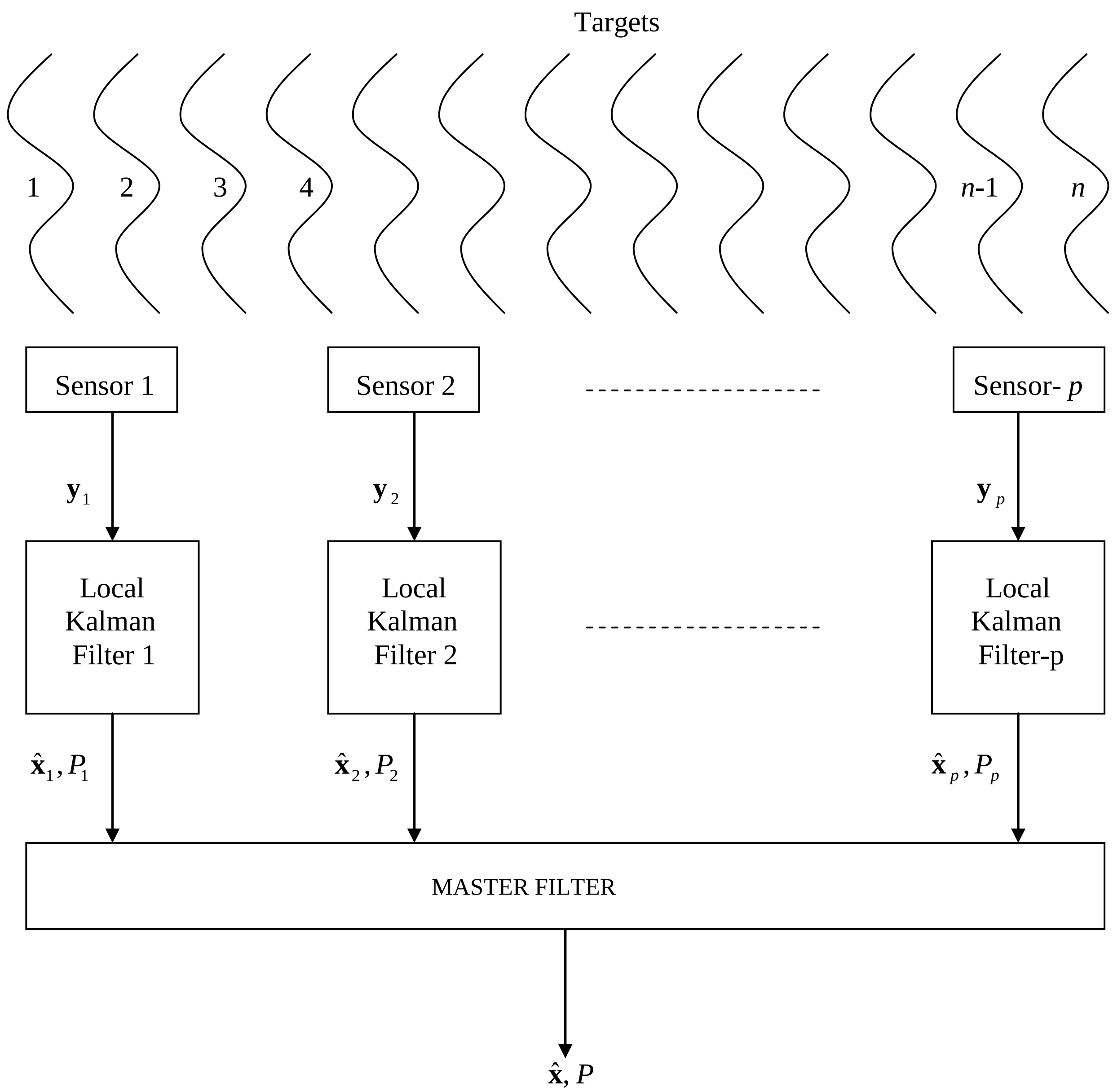

Fig. 5. Decentralized Kalman Filter.

Once each sensor estimates locally by the Kalman filters, the local outputs are transmitted to the master filter to finalize the track to track evaluations. These are considered by the assumption of data association process. The given equation (27) is decomposed of the locals of the system as;

$$\begin{aligned} x_{i\,j}(t+1) &= F_{i\,j}\; x_{i\,j}(t) + \omega_{i\,j}(t) \\ y_{i\,j}(t+1) &= H_{i\,j}\; x_{i\,j}(t) + \omega_{i\,j}(t) \end{aligned} \tag{28}$$

where,

$$\begin{aligned} i &= 1,2,\cdots, p = \text{number of sensors} \\ j &= 1,2,\cdots, n = \text{number of targets} \end{aligned} \tag{29}$$

Thus, by equation 28, i is the i-th sensor and j is the j-th target i-th sensor sees. All equations are assumed by the the coordinate system of the coordinator. Let write the each term of the equation 27 corresponding to the equation 28 respectively. Let start firstly with the state equations.

$$X(t+1) = \begin{bmatrix} x_{11}\mathbf{1}(t+1) \\ x_{21}\mathbf{1}(t+1) \\ \vdots \\ x_{n1}\mathbf{1}(t+1) \\ x_{12}\,\mathbf{2}(t+1) \\ x_{22}\,\mathbf{2}(t+1) \\ \vdots \\ x_{n2}\,\mathbf{2}(t+1) \\ x_{1p}\mathbf{p}(t+1) \\ x_{2p}\mathbf{p}(t+1) \\ \vdots \\ \vdots \\ x_{np}\mathbf{p}(t+1) \end{bmatrix};$$

$$W(t+1)=\begin{bmatrix} \omega_{11}\mathbf{1}(t+1) \\ \omega_{21}\mathbf{1}(t+1) \\ \vdots \\ \omega_{n1}\mathbf{1}(t+1) \\ \omega_{12}\mathbf{2}(t+1) \\ \omega_{22}\mathbf{2}(t+1) \\ \vdots \\ \omega_{n2}\mathbf{2}(t+1) \\ \omega_{1p}\mathbf{p}(t+1) \\ \omega_{2p}\mathbf{p}(t+1) \\ \vdots \\ \vdots \\ \omega_{np}\mathbf{p}(t+1) \end{bmatrix};$$

$$F=\begin{bmatrix}
F_{11}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & F_{21}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & F_{n1}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & F_{12}\mathbf{2} & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & F_{22}\mathbf{2} & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0\ \ddots & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & F_{n2}\mathbf{2} & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & F_{1p}\mathbf{p} & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & F_{2p}\mathbf{p} & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\ \ddots & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \quad F_{np}\mathbf{p}
\end{bmatrix} \tag{30}$$

As seen $X(t+1)$ and $W(t)$ matrices are formed by the field of views of the sensors. Each dashed zone in the matrices denotes the n targets seen by p sensors as x_{np}. The bold numbers as $\mathbf{1,2,\cdots,p}$ as in form of are used to indicate the targets seen by the related numbered sensors. Now let organize the expressions above by Equation $X(t+1)=FX(t)+W(t)$ as

$$\begin{bmatrix} x_{11}\mathbf{1}(t+1) \\ x_{21}\mathbf{1}(t+1) \\ \vdots \\ x_{n1}\mathbf{1}(t+1) \\ x_{12}\mathbf{2}(t+1) \\ x_{22}\mathbf{2}(t+1) \\ \vdots \\ x_{n2}\mathbf{2}(t+1) \\ x_{1p}\mathbf{p}(t+1) \\ x_{2p}\mathbf{p}(t+1) \\ \vdots \\ \vdots \\ x_{np}\mathbf{p}(t+1) \end{bmatrix}=\begin{bmatrix}
F_{11}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & F_{21}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & F_{n1}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & F_{12}\mathbf{2} & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & F_{22}\mathbf{2} & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0\ \ddots & 0 & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & F_{n2}\mathbf{2} & 0 & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & F_{1p}\mathbf{p} & 0 & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & F_{2p}\mathbf{p} & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\ \ddots & 0 \cdots 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \quad F_{np}\mathbf{p}
\end{bmatrix}\begin{bmatrix} x_{11}\mathbf{1}(t) \\ x_{21}\mathbf{1}(t) \\ \vdots \\ x_{n1}\mathbf{1}(t) \\ x_{12}\mathbf{2}(t) \\ x_{22}\mathbf{2}(t) \\ \vdots \\ x_{n2}\mathbf{2}(t) \\ x_{1p}\mathbf{p}(t) \\ x_{2p}\mathbf{p}(t) \\ \vdots \\ \vdots \\ x_{np}\mathbf{p}(t) \end{bmatrix}+\begin{bmatrix} \omega_{11}\mathbf{1}(t) \\ \omega_{21}\mathbf{1}(t) \\ \vdots \\ \omega_{n1}\mathbf{1}(t) \\ \omega_{12}\mathbf{2}(t) \\ \omega_{22}\mathbf{2}(t) \\ \vdots \\ \omega_{n2}\mathbf{2}(t) \\ \omega_{1p}\mathbf{p}(t) \\ \omega_{2p}\mathbf{p}(t) \\ \vdots \\ \vdots \\ \omega_{np}\mathbf{p}(t) \end{bmatrix} \tag{31}$$

Let Q_{ij} be the process noise matrix of j-th target seen by i-th sensor. Thus, its covariance is written as;

$$\mathrm{Cov}(\omega_{ij})=\begin{bmatrix} Q_j & 0 & 0 & 0 \\ 0 & Q_j & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & Q_j \end{bmatrix} \ ; \ i=1,2,\cdots,p \ ; \ j=1,2,\cdots,n \tag{32}$$

Or;

$$\mathrm{Cov}(W)=\begin{bmatrix} Q_1 & & & & & & & \mathbf{0} \\ & \ddots & & & & & & \\ & & Q_1 & & & & & \\ & & & Q_2 & & & & \\ & & & & \ddots & & & \\ & & & & & Q_2 & & \\ & & & & & & \ddots & \\ & & & & & & & Q_p \\ & & & & & & & \ddots \\ \mathbf{0} & & & & & & & Q_p \end{bmatrix} \tag{33}$$

Then, let form the steps for the output equations

$$Y(t)=\begin{bmatrix} y_{11}\mathbf{1}(t) \\ y_{21}\mathbf{1}(t) \\ \vdots \\ y_{n1}\mathbf{1}(t) \\ y_{12}\mathbf{2}(t) \\ y_{22}\mathbf{2}(t) \\ \vdots \\ y_{n2}\mathbf{2}(t) \\ y_{1p}\mathbf{p}(t) \\ y_{2p}\mathbf{p}(t) \\ \vdots \\ \vdots \\ y_{np}\mathbf{p}(t) \end{bmatrix};$$

$$V(t)=\begin{bmatrix} v_{11}\mathbf{1}(t) \\ v_{21}\mathbf{1}(t) \\ \vdots \\ v_{n1}\mathbf{1}(t) \\ v_{12}\mathbf{2}(t) \\ v_{22}\mathbf{2}(t) \\ \vdots \\ v_{n2}\mathbf{2}(t) \\ v_{1p}\mathbf{p}(t) \\ v_{2p}\mathbf{p}(t) \\ \vdots \\ \vdots \\ v_{np}\mathbf{p}(t) \end{bmatrix};$$

$$H=\begin{bmatrix} H_{11}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & H_{21}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & H_{n1}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & H_{12}\mathbf{2} & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & H_{22}\mathbf{2} & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 \ddots 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & H_{n2}\mathbf{2} & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{1p}\mathbf{p} & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{2p}\mathbf{p} & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \ddots & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \;\; H_{np}\mathbf{p} \end{bmatrix} \tag{34}$$

Let organize it by equation $Y(t)=HX(t)+V(t)$

$$\begin{bmatrix} y_{11}\mathbf{1}(t) \\ y_{21}\mathbf{1}(t) \\ \vdots \\ y_{n1}\mathbf{1}(t) \\ y_{12}\mathbf{2}(t) \\ y_{22}\mathbf{2}(t) \\ \vdots \\ y_{n2}\mathbf{2}(t) \\ y_{1p}\mathbf{p}(t) \\ y_{2p}\mathbf{p}(t) \\ \vdots \\ \vdots \\ y_{np}\mathbf{p}(t) \end{bmatrix} = \begin{bmatrix} H_{11}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & H_{21}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & H_{n1}\mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & H_{12}\mathbf{2} & 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & H_{22}\mathbf{2} & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 \ddots 0 & 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & H_{n2}\mathbf{2} & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{1p}\mathbf{p} & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_{2p}\mathbf{p} & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \ddots & 0 \cdots 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \;\; H_{np}\mathbf{p} \end{bmatrix} \begin{bmatrix} x_{11}\mathbf{1}(t) \\ x_{21}\mathbf{1}(t) \\ \vdots \\ x_{n1}\mathbf{1}(t) \\ x_{12}\mathbf{2}(t) \\ x_{22}\mathbf{2}(t) \\ \vdots \\ x_{n2}\mathbf{2}(t) \\ x_{1p}\mathbf{p}(t) \\ x_{2p}\mathbf{p}(t) \\ \vdots \\ \vdots \\ x_{np}\mathbf{p}(t) \end{bmatrix} + \begin{bmatrix} v_{11}\mathbf{1}(t) \\ v_{21}\mathbf{1}(t) \\ \vdots \\ v_{n1}\mathbf{1}(t) \\ v_{12}\mathbf{2}(t) \\ v_{22}\mathbf{2}(t) \\ \vdots \\ v_{n2}\mathbf{2}(t) \\ v_{1p}\mathbf{p}(t) \\ v_{2p}\mathbf{p}(t) \\ \vdots \\ \vdots \\ v_{np}\mathbf{p}(t) \end{bmatrix} \tag{35}$$

The expressions in $X(t+1)$ and $Y(t)$ are for the coordinate system of the coordinator. Let R_{ij} be the measurement noise matrix of j-th target seen by i-th sensor. Thus, its covariance is written as;

$$\mathrm{Cov}(v_{ij})=\begin{bmatrix} R_j & 0 & 0 & 0 \\ 0 & R_j & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & R_j \end{bmatrix} \quad ; \; i=1,2,\cdots,p \; ; \; j=1,2,\cdots,n \tag{36}$$

Or;

$$\mathrm{Cov}(V)=\begin{bmatrix} R_1 & & & & & & & & \mathbf{0} \\ & \ddots & & & & & & & \\ & & R_1 & & & & & & \\ & & & R_2 & & & & & \\ & & & & \ddots & & & & \\ & & & & & R_2 & & & \\ & & & & & & \ddots & & \\ & & & & & & & R_p & \\ & & & & & & & & \ddots \\ \mathbf{0} & & & & & & & & R_p \end{bmatrix} \tag{37}$$

<u>Mapping matrix</u>

In this work, a mapping matrix is presented using a new approach to achieve track to track correlation of targets. As stated earlier, in order to solve the track-to track correlation problems, D_i matrix which arose in decentralized filtering indicates the targets tracked by each individual local sensor. Let recall the general form of the mapping matrix defined in the global model.

The D_i arises in decentralized filtering which indicates the targets are tracked by each individual local sensor. The fusion algorithm in the central processor is applied to combine the local sensor track results with this matrix. Such a matrix is defined as follows:

$$D_i=[D_i^{jk}] \;,\; j=1,2,\cdots,n_i \quad ; \quad k=1,2,\cdots,n \tag{38}$$

where M_i^{jk} is an n by n block that indicates which targets are seen by the *i*-th sensor, and the *n* is the order of state vector for each target.

Here the global mapping matrix is developed as the modified case of $D_i(t)$. The new mapping matrix was derived by the hypothesizes as a row matrix in form of D_i^j, where *j* denotes the hypothesis number and *i* is the number of sensor. The presented mapping matrix was conducted for the track to track correlation process. as the following. Let make some hypothesis for track-to-track correlation procedure.

Hypothesis 1 : HIP_1 : If all targets are distinct

Let process the equations by HIP_1. For each sensor (*i*-th sensor), $i=1,2,\cdots,p$ let write the measured equation.

$$\underbrace{\begin{bmatrix} y_{1i}(t) \\ y_{2i}(t) \\ \vdots \\ y_{ni}(t) \end{bmatrix}}_{Y_i(t)} = \underbrace{\begin{bmatrix} H_{1i} & 0 & 0 & 0 \\ 0 & H_{2i} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & H_{ni} \end{bmatrix}}_{H_i(t)} \underbrace{\begin{bmatrix} x_{1i}(t) \\ x_{2i}(t) \\ \vdots \\ x_{ni}(t) \end{bmatrix}}_{X_i(t)} + \underbrace{\begin{bmatrix} v_{1i}(t) \\ v_{2i}(t) \\ \vdots \\ v_{ni}(t) \end{bmatrix}}_{V_i(t)} \tag{39}$$

In this case, If all targets were distinct, the targets seen by the first sensor would be unchanged and Equation (39) would be as

$$\begin{bmatrix} y_{11}(t) \\ y_{21}(t) \\ \vdots \\ y_{n1}\mathbf{1}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} H_{11} & 0 & 0 & 0 \\ 0 & H_{21} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & H_{n1}\mathbf{1} \end{bmatrix}}_{H_1(t)} \begin{bmatrix} x_{11}(t) \\ x_{21}(t) \\ \vdots \\ x_{n1}\mathbf{1}(t) \end{bmatrix} + \begin{bmatrix} v_{11}(t) \\ v_{21}(t) \\ \vdots \\ v_{n1}\mathbf{1}(t) \end{bmatrix} \tag{40}$$

The general representation of this case would be as

$$Y_1(t) = H_1 X_1(t) + V_1(t) \tag{41}$$

For global case, the coordinating sensor with all targets would be measured as

$$Y_1(t) = H_1 \underbrace{\begin{bmatrix} I & : & 0 & : & 0 & : & \cdots & : & 0 \end{bmatrix}}_{D_1^1} \begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_p(t) \end{bmatrix} + V(t) \tag{42}$$

Or;

$$Y_1(t) = H_1 \underbrace{\begin{bmatrix} I_n & 0 & 0 & \cdots & 0 & & 0 & 0 & 0 & \cdots & 0 & & & & & & & & \\ 0 & I_n & 0 & \cdots & 0 & & 0 & 0 & 0 & \cdots & 0 & & & & & & & & \\ 0 & 0 & I_n & \cdots & 0 & : & 0 & 0 & 0 & \cdots & 0 & : & 0 & : & \cdots & : & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \vdots & \vdots & \vdots & & & & & & & & \\ 0 & 0 & 0 & \cdots & I_n & & 0 & 0 & 0 & 0 & 0 & & & & & & & & \end{bmatrix}}_{D_1^1} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \\ \vdots \\ X_p(t) \end{bmatrix} + V(t) \tag{43}$$

These equations for the second sensor are derived from HIP_1. By this hypothesis, D_1^1 is a row matrix which indicates the targets seen only by the first sensor by HIP_1. In fact, D_i^j is a row matrix, j denotes the hypothesis number and i is the number of sensor. Items of the row matrix consist of n by n dimensional block matrices of the targets seen by the sensors. Equation (43) reflects the targets seen only by the first sensor. As the first sensor does not sees the targets of the other sensors, the items in D_1^1 row matrix belonged to the other sensors consist of zero blocks. Because, by the assumed hypothesis, the targets are distinct and the sensors does not see the targets out of the view of their fields. By HIP_1 Equation (41) can be written with respect to the global state as the following

$$\begin{aligned} Y_1(t) &= C_1^1 \; X_1(t) + V_1(t) \\ &= H_1 \; D_1^1 X_1(t) + V_1(t) \end{aligned} \tag{44}$$

where,

$$C_1^1 = H_1 \; D_1^1 \tag{45}$$

As noted, the time varying mapping matrix, $D(t)$ defined earlier, now is in a time invariant form as D_i^k due to the assumption of the data association process. If we take D_i^k, where k denotes the hypothesis number and \$i\$ denotes the sensor number. For example D_2^1 indicates the targets seen by the second sensor by the first hypothesis. Hence, the following can be written.

$$C_i^k = H_i \; D_i^k \tag{46}$$

In general C_i^k can be written as C_i as mapping matrix which is defined to associate the local sensors with the central processors, which offers the track-to track correlation information. Thus, the general view of the mapping matrix is as

$$C_i = H_i \; D_i \tag{47}$$

where D_i reflects which of the targets are observed by the i-th sensor. Now considering HIP_1, let make the same steps for the second sensor. If all targets were distinct, the equation for the second sensor would be as

$$\begin{bmatrix} y_{121}(t) \\ y_{22}(t) \\ \vdots \\ y_{n2}\mathbf{2}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} H_{12} & 0 & 0 & 0 \\ 0 & H_{22} & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & H_{n2}\mathbf{2} \end{bmatrix}}_{H_2(t)} \begin{bmatrix} x_{12}(t) \\ x_{22}(t) \\ \vdots \\ x_{n2}\mathbf{2}(t) \end{bmatrix} + \begin{bmatrix} v_{12}(t) \\ v_{22}(t) \\ \vdots \\ v_{n2}\mathbf{2}(t) \end{bmatrix} \tag{48}$$

The general representation of this case would be as

$$Y_2(t) = H_2 \; X_2(t) + V_2(t) \tag{49}$$

For global case, the coordinating sensor with all targets would be measured as

$$\underbrace{\begin{bmatrix} Y_1(t) \\ Y_2(t) \\ \vdots \\ Y_p(t) \end{bmatrix}}_{Y(t)} = \underbrace{\begin{bmatrix} H_1 & 0 & 0 & 0 \\ 0 & H_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & H_p \end{bmatrix}}_{H} \underbrace{\begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_p(t) \end{bmatrix}}_{X(t)} + \underbrace{\begin{bmatrix} V_1(t) \\ V_2(t) \\ \vdots \\ V_n(t) \end{bmatrix}}_{V(t)} \tag{50}$$

By HIP_1, Equation (49) can be written with respect to the global state as the following

$$Y_2(t) = H_2 \underbrace{\begin{bmatrix} 0 & : & I & : & 0 & : & \cdots & : & 0 \end{bmatrix}}_{D_2^1} \begin{bmatrix} X_1(t) \\ X_2(t) \\ \vdots \\ X_p(t) \end{bmatrix} + V(t) \tag{51}$$

As can be seen, D_2^1 is a row matrix which indicates the targets seen only by the second sensor. In fact, we know also that D_2^1 is a row matrix whose items consist of n by n dimensional block matrices of the targets seen by the sensors. As realized in D_2^1 row matrix, except the second senor, items of the other sensors are empty (zero). Because the second sensor does not see the targets seen by the other sensors. The row matrix shows only the targets seen by the second sensor. Because all sensors see distinct targets by HIP_1.

For simplicity of exposition assume that all $X_j(t)$ have dimension n. Then *I* in (48) is of the form as

$$\mathrm{I} = \begin{bmatrix} I_n & 0 & 0 & 0 \\ 0 & I_n & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & I_n \end{bmatrix} \tag{52}$$

In other words,

$$Y_2(t) = H_2 X_2(t) + V_2(t) \tag{53}$$

Or;

$$Y_2(t) = C_2^1 X(t) + V(t) \tag{54}$$

where

$$C_2^1 = H_2 D_2^1 \tag{55}$$

The expanded case of Equation (51) can be written as

$$Y_2(t) = H_2 \underbrace{\left[\begin{array}{ccccccccccccccccc} 0 & 0 & 0 & \cdots & 0 & & I_n & 0 & 0 & \cdots & 0 & & & & & & \\ 0 & 0 & 0 & \cdots & 0 & & 0 & I_n & 0 & \cdots & 0 & & & & & & \\ 0 & 0 & 0 & \cdots & 0 & : & 0 & 0 & I_n & \cdots & 0 & : & 0 & : & \cdots & : & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots & & & & & & \\ 0 & 0 & 0 & \cdots & 0 & & 0 & 0 & 0 & 0 & I_n & & & & & & \end{array}\right]}_{D_2^1} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \\ \vdots \\ X_p(t) \end{bmatrix} + V(t) \tag{56}$$

As stated, Equation (56) indicates that the targets are observed only by the second sensor. For example, this equation does not show that the first target is also seen by the first sensor. By HIP_1, the first sensor cannot observe the targets seen by the second sensor. In other words, the first and second sensor does not see the first or the same target simultaneously.

Now, let consider the second hypothesis, HIP_2 as

HIP_2 : The targets with the nearest trajectories are the same.

Let assume HIP_2 that the first target seen by both first sensor and second sensor is same. Then, as the equations of the first and the other sensors remain unchanged, the equation of the second sensor is modified as

$$Y_2(t) = H_2 \underbrace{\left[\begin{array}{ccccccccccccccccc} I_n & 0 & 0 & \cdots & 0 & & 0 & 0 & 0 & \cdots & 0 & & & & & & \\ 0 & 0 & 0 & \cdots & 0 & & 0 & I_n & 0 & \cdots & 0 & & & & & & \\ 0 & 0 & 0 & \cdots & 0 & : & 0 & 0 & I_n & \cdots & 0 & : & 0 & : & \cdots & : & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots & & & & & & \\ 0 & 0 & 0 & \cdots & 0 & & 0 & 0 & 0 & 0 & I_n & & & & & & \end{array}\right]}_{D_2^1} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \\ \vdots \\ X_p(t) \end{bmatrix} + V(t) \tag{57}$$

In other words,

$$Y_2(t) = C_2^2 X(t) + V(t) \tag{58}$$

where

$$C_2^2 = H_2 D_2^2 \tag{59}$$

Thus,

$$Y_2(t) = H_2 D_2^2 X(t) + V(t) \tag{60}$$

If consider (57) it will be realized that, the second sensor sees the first target seen by the first sensor. In other words, both the first and second sensors observe the first target simultaneously. Whereas Equation (56) does not denote any I_n block for the first sensor, the same equation consists of I_n blocks which indicates the distinct targets of the second sensor. From D_2^1 parts of Equations (56) and (57), the following (first) column

$$\begin{bmatrix} I_n & : & 0 & : & 0 & : & \cdots & : & 0 \end{bmatrix} \tag{61}$$

in Equation (56) of D_2^1 for the second sensor is replaced by the following (first) column (zero column)

$$\begin{bmatrix} 0 & : & 0 & : & 0 & : & \cdots & : & 0 \end{bmatrix} \tag{62}$$

in Equation (57}) of D_2^1 for the first sensor. Hence, when considers overall Equation (57), it provides two information. Firstly it denotes all targets seen by the second sensor. In addition, it emphasis on the first target which is observed jointly by the first sensor. As a result, when check the Equation (57) or its part of D_2^1, distinct and joint targets shared by the other sensors can be observed. Maybe (57) could alternatively be written as in the following form

$$Y_2(t) = H_2 \underbrace{\left[\begin{array}{ccccccccccccccccc} I_n & 0 & 0 & \cdots & 0 & & I_n & 0 & 0 & \cdots & 0 & & & & & & \\ 0 & 0 & 0 & \cdots & 0 & & 0 & I_n & 0 & \cdots & 0 & & & & & & \\ 0 & 0 & 0 & \cdots & 0 & : & 0 & 0 & I_n & \cdots & 0 & : & 0 & : & \cdots & : & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & & \vdots & \vdots & \vdots & \ddots & \vdots & & & & & & \\ 0 & 0 & 0 & \cdots & 0 & & 0 & 0 & 0 & 0 & I_n & & & & & & \end{array}\right]}_{D_2^1} \begin{bmatrix} X_1(t) \\ X_2(t) \\ X_3(t) \\ \vdots \\ X_p(t) \end{bmatrix} + V(t) \tag{63}$$

This can be evaluated as the slightly modified form of D_2^1. Equation (57) seems more efficient compared to the the last equation (63) in terms of the practical. Because, if D_2^1 in (57) is assumed a look-up table, it can be able to give a knowledge on both seen the distinct and joint targets of the second sensor. The equation (57) provides this by an additional information on the sensor (s), which observe the related joint target.

Other possible hypothesis $HIP_3, HIP_4, \cdots, HIP_N$ can be represented as described above for HIP_1 and HIP_2, by proper choices of C_j^k, D_j^k matrices for those sensors which see the same target. Thus, each hypothesis (say *k*-th) a global system representation for data as

$$\begin{aligned} X(t+1) &= FX(t) + W(t) \\ Y_k(t) &= C_j^k X(t) + V(t) \end{aligned} \tag{64}$$

where

$$C_j^k = H_j D_j^k \tag{65}$$

Now we can apply the formulas for decentralized Kalman filtering to obtain a filter for each global state for each hypothesis.

For each such HIP_k let $\tilde{\theta}_k$ be the corresponding global system representation (parameters). For each $\tilde{\theta}_k$ (hypothesis), check the innovations (residual) process γ_i^k for the corresponding Kalman filter for whiteness. If $\tilde{\theta}_k$ represents the actual data (i.e., if it is correct) then the corresponding residual process must be white noise. Using tests such as chi-square test and/or several developed fault detection techniques, we can eliminate some of these $\tilde{\theta}_k$'s (those for which the innovations process does not pass the whiteness test).

Let $\theta_i, \cdots, \theta_M$ be the hypothesis which pass the whiteness test. Now we can use wellknown recursive formulas for estimating.

$$P(\theta_k \mid Y(t), Y(t-1), \cdots, Y(0)) \tag{66}$$

as well as

$$P(Y(t), \cdots, Y(0) \mid \theta_k) \tag{67}$$

The θ_k which maximizes (66) and (67) gives us the maximum a posteriori (maximum likelihood) estimate of true value of θ_k 's (i.e., the true hypothesis). See [Ref:20}, Chapter 8] for details.

4. Conclusion

Physical systems are often subjected to unexpected changes, such as component failures and variations in operating condition, that tend to degrade overall system performance. These changes called as “failures”. The failures in each case are a defect to reduce a system's effectiveness [47-49]. In order to maintain a high level of performance, it is important that failures be promptly detected and identified so that appropriate remedies can be applied. Over the past decade numerous approaches to the problem of failure detection and identification (FDI) in dynamical systems have been developed; detection filters, the generalized likelihood ratio (GLR) method and the multiple model methods are some examples [44-49]. Ones that fail are discarded first.

As for the multisensor fusion problems, several track to track correlation hypotheses are first tested using failure detection techniques. Among those which do not fail, we choose that hypothesis to be the correct one by using maximum likelihood and/or maximum a posteriori estimation techniques.

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