A new seven-term chaotic attractor and its hyperchaos

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In this paper a new chaotic system reported. Some basic dynamical properties of the new attractor are demonstrated in terms of equilibria, Jacobian matrices, Lyapunov exponents, chaotic waveform in time domain, continuous frequency spectrum and its hyperchaos case. The new system and its hyperchaos case are examined in Matlab-Simulink. The new system has seven terms, two quadratic nonlinearities and has rich dynamic behaviours.

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1. Introduction

Chaos has been found to be useful, or has great potential to be useful, in many disciplines, for example, information processing, collapse prevention of power systems, high-performance circuits and devices, thorough liquid mixing with low power consumption [1]. In 1963, Lorenz found the first chaotic attractor in a three-dimensional autonomous system when he studied the atmospheric convection [2], In 1976, Rössler carried out an important work, which rekindled the interest in low dimensional dissipative dynamical systems [3]. In 1979, Rössler himself proposed an even simpler (algebraic) system [4]. Sprott embarked upon an extensive search [5] for autonomous three dimensional chaotic systems with fewer than seven terms in the right hand side of the model equations. Sprott considered general three dimensional ordinary differential equations with quadratic nonlinearities. Using the numerical search several thousand chaotic cases were found. 19 cases (Labeled by 'A' to 'S') appear to be distinct in the sense that there is no obvious transformation of one to another. In these 19 ('A' to 'S') cases, 'A' to 'E' (five) have five terms and two nonlinearities while cases 'F' to 'S' (fourteen) have six terms and one nonlinearity in the right hand side. Finally in the conclusive paragraph Sprott stated that "Method employed can not guarantee that these are the simplest chaotic systems of ordinary differential equations or that all the chaotic systems of three-dimensional ordinary differential equations with five terms and two quadratic nonlinearities or with six terms and one quadratic nonlinearity have been discovered. However the cases with five terms appeared early and often in the search, and it is likely they have all been found. New cases with six terms were still being found and thus additional such cases probably exist". Recently, there has been increasing interest in exploiting chaotic dynamics in engineering applications, where some attention has been focused on effectively creating chaos via simple physical systems such as electronic circuits [6]-[17].

Motivated by these works, this article introduces one more simple three-dimensional quadratic autonomous system. The aim of this letter is to present a simple, interesting and complex three-dimensional chaotic system, which can depict complex 1-scroll chaotic attractors simultaneously. Section 2 explains the new chaotic system. In this section, the simulation result of the new system using Matlab-Simulink modelling (Fig. 1.h) is obtained. In Section 3 some basic properties and finally in Section 4 forming mechanism of this new chaotic attractor structure, hyperchaos behaviour of the new system are given respectively.

2. A new seven term chaotic system

Based on the chaotification analysis [5] the third differential equation of the Sprott S case may be added with a term $u_1 = -b * y^2$, i.e. $\dot{z} = 1 + x - b * y^2$. A new chaotic system is therefore expressed as a set of two first order and one second order, autonomous, ordinary differential equations with seven terms as follows:

$$\begin{aligned} \dot{x} &= -x - a * y \\ \dot{y} &= x + z^2 \\ \dot{z} &= 1 + x - b * y^2 \end{aligned} \tag{1}$$

where $a = 4, b = 0.2$. Using MATLAB program, the numerical simulation have been completed and displayed in Figure 1(a)-(h), three dimensional, x-y, x-z and y-z phase portraits, x(t) waveform, y(t) waveform, z(t) waveform and the MATLAB-Simulink modelling, respectively.

The initial conditions are selected as $(x_0, y_0, z_0) = (0.67, 0.0067, 0.89)$.

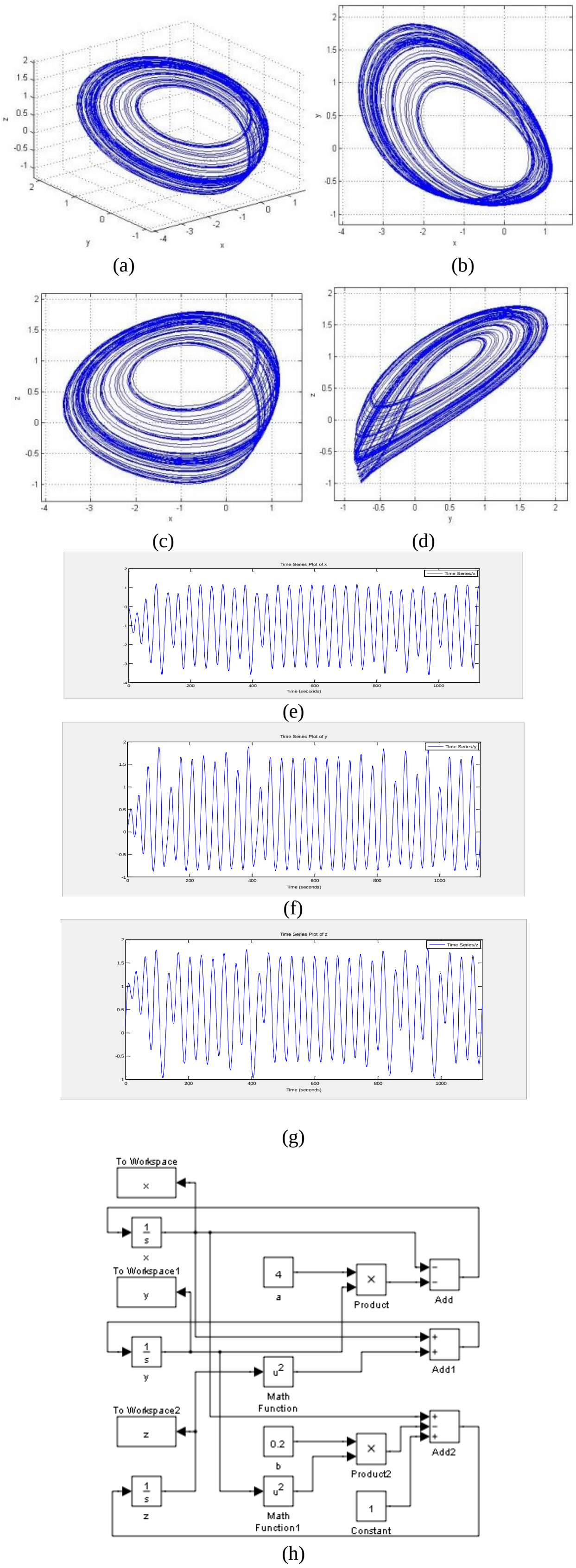


Fig. 1. Phase portraits, time series and simulink model of a new seven-term chaotic attractor, (a) x-y-z, (b) x-y, (c) y-z, (d) y-z, (e) x(t) waveform, (f) y(t) waveform, (g) z(t) waveform and (h) The MATLAB-Simulink modelling of the new system.

3. Some basic properties of the new chaotic system

In this section, equilibria, Jacobian Matrix, Lyapunov exponents and being a dissipative system of the new chaotic attractor are analized as follows,

3.1. Equilibria

The Equilibria of the new system (1) can be found using,

$$-x - 4 * y = 0$$
$$x + z^2 = 0$$
$$1 + x - 0.2 * y^2 = 0.$$

The system has four equilibrium points, i.e.

$$E_1 = (-0.99, 0.25, 0.99)$$
$$E_2 = (-0.99, 0.25, -0.99)$$
$$E_3 = (80.99, -20.25, 9\mathrm{i})$$
$$E_4 = (80.99, -20.25, -9\mathrm{i}).$$

3.2. Jacobian matrice

For equilibrium $E_1 = (-0.99, 0.25, 0.99)$, system (1) are linearized and the Jacobian matrix (2) is defined as,

$$J_1 = \begin{bmatrix} -1 & -a & 0 \\ 1 & 0 & 2*z \\ 1 & -2*b*y & 0 \end{bmatrix} = \begin{bmatrix} -1 & -4 & 0 \\ 1 & 0 & 1.98 \\ 1 & -0.1 & 0 \end{bmatrix} \tag{2}$$

By using $|\lambda I - J_1| = 0$, the resulting eigenvalues of J_1 are obtained as follows where λ_1 is a negative real number, λ_2 and λ_3 are a pair of complex conjugate eigenvalues with positive real parts. Consequently, the equilibrium E_1 is a saddle-focus point and the new system (1) is unstable at E_1 equilibrium point.

$$\lambda_1 = -1.58, \lambda_2 = 0.29 + 2.24i, \lambda_3 = 0.29 - 2.24i$$

For equilibrium $E_2 = (-0.99, 0.25, -0.99)$, the resulting eigenvalues are

$$\lambda_1 = 1.24, \lambda_2 = -1.12 + 2.31i, \lambda_3 = -1.12 - 2.31i.$$

For equilibrium $E_3 = (80.99, -20.25, 9\mathrm{i})$, the resulting eigenvalues are

$$\lambda_1 = -0.51 + 0.01i, \lambda_2 = 8.18 + 8.64i,$$
$$\lambda_3 = -8.68 - 8.65i.$$

And finally for equilibrium $E_3 = (80.99, -20.25, -9\mathrm{i})$, the resulting eigenvalues are

$$\lambda_1 = -0.51 - 0.01i, \lambda_2 = 8.18 - 8.64i,$$
$$\lambda_3 = -8.68 + 8.65i,$$

respectively.

3.3. Lyapunov exponents

It is known that Lyapunov exponents are the average exponential rates of divergence or convergence of nearby trajectories in the phase space. Any system containing at least one positive Lyapunov exponent is defined to be chaotic [18]. The Lyapunov exponents of the system (1) are displayed in Fig. 2 and found to be

$$L_1 = 0.10482,\ L_2 = 0 \ \text{ and } L_3 = -1.1062.$$

In addition, the Lyapunov dimension (3) of the system (1) is fractional as described by

$$D_L = j + \frac{1}{|L_{j+1}|}\sum_{i=1}^{j} L_i = 2 + \frac{L_1+L_2}{|L_3|} = 2.095. \qquad (3)$$

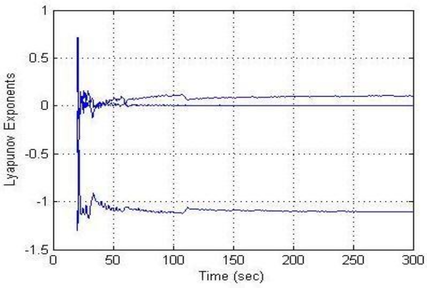


Fig. 2. Lyapunov exponents of the new chaotic system.

The fractal nature of an attractor does not merely imply non-periodic orbits; it also causes nearby trajectories to diverge. As all strange attractors, orbits that are initiated from different initial conditions soon reach the attracting set, but two nearby orbits do not stay close to each other [12]. They soon diverge and follow totally different paths in the attractor. Therefore there is really chaos in this system.

3.4 A dissipative system and existence of the attractor

For the divergence of flow (4) of the new system (1), we can obtain,

$$\nabla.V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = p = -1 \qquad (4)$$

As $p < 0$, the system (1) is a dissipative system with an exponential rate of contraction as,

$$\frac{dV}{dt} = e^{p} = e^{-1}$$

In the new system (1), a volume element V_0 is apparently contracted by the flow into a volume element,

$$V_0 e^{pt} = V_0 e^{-t}$$

in time t. This means that each volume containing the trajectory of this dynamical system shrinks to zero as $t \to \infty$ at an exponential rate. So all this new dynamical system (1) orbits are eventually confined to a specific subset that have zero volume, the asymptotic motion settles on to an attractor of the new dynamics system (1)[18].

4. Forming mechanism structure of this new chaotic attractor system

Our new chaotic system (1) is investigated the forming mechanism structure by changing “b” parameter in 4.1 as follows. In 4.2 hyperchaos structure of the new system (1) is studied.

4.1. Case1: For $b = 0.2$,

In this case, equilibrium points of the systems (1) was given in section 3 as shown in Fig 1(a).

Case2: If $b = 0.6$, (Fig. 3.a) than the system (1) has four equilibrium points, i.e.

$$E_1 = (-0.97, 0.24, 0.98)$$
$$E_2 = (-0.97, 0.24, -0.98)$$
$$E_3 = (27.63, -6.91, 5.26\text{i})$$
$$E_4 = (27.63, -6.91, -5.26\text{i}).$$

For equilibrium $E_1 = (-0.97, 0.24, 0.98)$ the resulting eigenvalues are

$$\lambda_1 = -1.55,\ \lambda_2 = 0.28 + 2.31i,\ \lambda_3 = 0.28 - 2.31i.$$

For equilibrium $E_2 = (-0.97, 0.24, -0.98)$ the resulting eigenvalues are

$$\lambda_1 = 1.30,\ \lambda_2 = -1.15 + 2.26i,\ \lambda_3 = -1.15 - 2.26i.$$

For equilibrium $E_3 = (27.63, -6.91, 5.26\text{i})$ the resulting eigenvalues are

$$\lambda_1 = 6.22 + 6.73i,\ \lambda_2 = -0.52 + 0.02i,$$
$$\lambda_3 = -6.71 - 6.76i.$$

And finally for equilibrium $E_4 = (27.63, -6.91, -5.26\text{i})$ the resulting eigenvalues are

$$\lambda_1 = 6.22 - 6.73i,\ \lambda_2 = -0.52 - 0.02i,$$
$$\lambda_3 = -6.71 + 6.76i,$$

respectively.

The Lyapunov exponents of this case are displayed in Fig. 3.b and found to be

$$L_1 = 0,\ L_2 = -0{,}15314 \ \text{ and } L_3 = -0.84614.$$

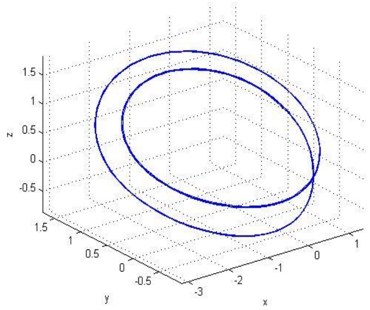


Fig. 3.a. The new system with b=0.6.

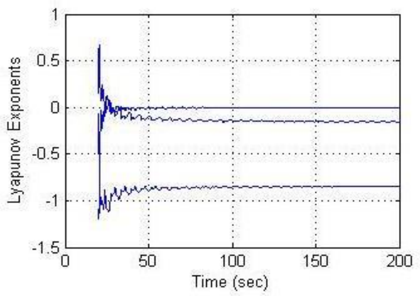


Fig. 3.b. Lyapunov Exponents.

Case3: If $b = 1.9$, (Fig 4.a) than the system (1) has four equilibrium points, i.e.

$$E_1 = (-0.90, 0.23, 0.95)$$
$$E_2 = (-0.90, 0.23, -0.95)$$
$$E_3 = (9.32, -2.33, 3.05\mathrm{i})$$
$$E_4 = (9.32, -2.33, -3.05\mathrm{i}).$$

For equilibrium $E_1 = (-0.90, 0.23, 0.95)$ the resulting eigenvalues are

$$\lambda_1 = -1.53, \lambda_2 = 0.26 + 2.29i, \lambda_3 = 0.26 - 2.29i.$$

For equilibrium $E_2 = (-0.90, 0.23, -0.95)$ the resulting eigenvalues are

$$\lambda_1 = 1.27, \lambda_2 = -1.14 + 2.25i, \lambda_3 - 1.14 - 2.25i.$$

For equilibrium $E_3 = (9.32, -2.33, 3.05\mathrm{i})$ the resulting eigenvalues are

$$\lambda_1 = 1.91 + 3.35i, \lambda_2 = 0.40 - 0.10i,$$
$$\lambda_3 = -3.31 - 3.24i.$$

And finally for equilibrium $E_4 = (9.32, -2.33, -3.05\mathrm{i})$ the resulting eigenvalues are

$$\lambda_1 = 1.91 - 3.35i, \lambda_2 = 0.40 + 0.10i,$$
$$\lambda_3 = -3.31 + 3.24i,$$

respectively.

The Lyapunov exponents of this case are displayed in Fig. 4.b and found to be

$$L_1 = 0, L_2 = -0{,}19954 \text{ and } L_3 = -0.7989$$

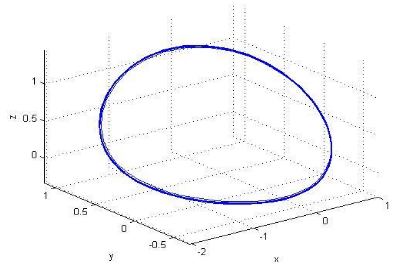


Fig. 4.a. The new system with b=1.9 .

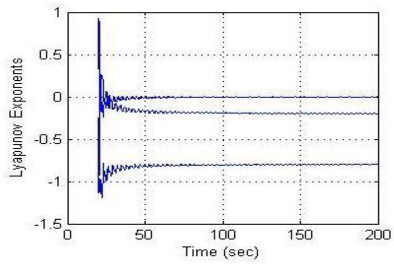


Fig. 4.b. Lyapunov Exponents.

4.2 Hyperchaos case of the new system

The new hyperchaotic equations (5) are obtained from the new system (1) with initial conditions $(x_0, y_0, z_0, w_0) = (0.67, 0.0067, 0.89, 0.2)$ as follows:

$$\begin{aligned} \dot{x} &= -x - a*y + c*w \\ \dot{y} &= x + z^2 \\ \dot{z} &= 1 + x - b*y^2 \\ \dot{w} &= k*x \end{aligned} \qquad (5)$$

where $c = 0.2, k = 0.2$.

Using MATLAB program, the numerical simulation have been completed and displayed x-y, x-z, y-z and x-y-z phase portraits and the MATLAB-Simulink model in Figs. 5(a)-(e), respectively.

The new hyperchaotic system (5) has two equilibrium points, i.e.

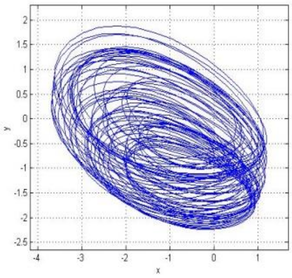
$$E_1 = (0, 0.73, 0, 14.51)$$
$$E_2 = (0, -0.73, 0, -14.51)$$

Jacobian matrix of the new hyperchaotic system (6) is defined as,

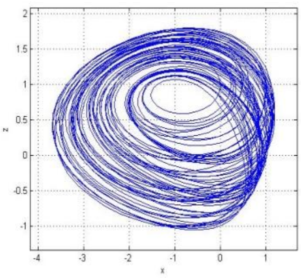
$$J = \begin{bmatrix} -1 & -a & 0 & 0 \\ 1 & 0 & 2*z & 0 \\ 1 & -2*b*y & 0 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix} \qquad (6)$$

And the eigenvalues E_1, E_2 are equal and as follows:

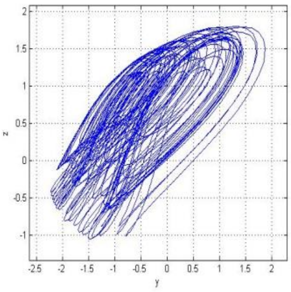
$$\lambda_1 = 0, \lambda_2 = 0.2, \lambda_3 = 0.50 + 1.94i, \lambda_4 = 0.50 - 1.94i.$$



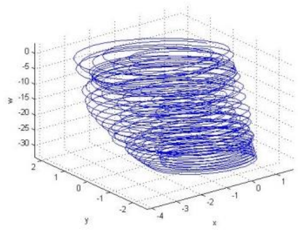
(a)



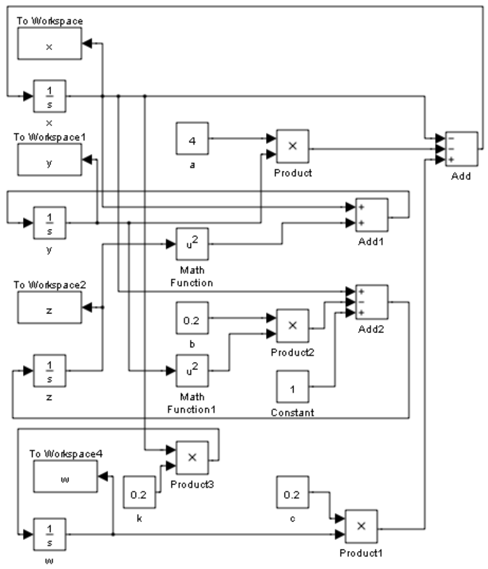
(b)



(c)



(d)



(e) The Matlab-Simulink modelling of the new hyperchaos system

Fig. 5. Phase portraits and simulink model of the new hyperchaotic system (a) x-y, (b) x-z, (c) y-z and (d) x-y-z (e)The Matlab-Simulink model.

5. Conclusions

This article introduces a new simple, three-dimensional, quadratic and autonomous chaotic system, which can generate complex 1-scroll chaotic attractors simultaneously. Our investigation was completed using a combination of theoretical analysis and simulation. This new attractors proposed can be also realized with an electronic circuit and have potential for communication. These new attractors and their forming mechanism need further to study and explore. The our new chaotic and hyperchaotic systems have a small margin for parameter varying for easy analysing, as shown in the phase portraits of figure 1 and figure 5, respectively. The simulation results were produced using Matlab-Simulink programs

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