OPTOELECTRONICS AND ADVANCED MATERIALS – RAPID COMMUNICATIONS Vol. 4, No. 7, July 2010, p. 909 - 915

A complete band gap in one-dimensional photonic crystal containing meta-materials

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A complete band gap in one-dimensional photonic crystal (1D-PC) containing meta-material is investigated. The average zero-*n* gap and Bragg gap are appeared in 1D-PC due to the interference in periodic structure of medium with positive and negative refractive indices. These band gaps are strongly sensitive to the separation between the electric and magnetic plasma frequencies. The average zero-*n* gap of the structure observes insensitive to incident angles and polarizations. But the Bragg-gap shifts towards higher frequency with different angle of incidences due to the change in the optical path through each layer. The transmittance of the structure for TE and TM-modes shows a complete band gap owing to the mechanism of the average zero-*n* when the separation between electric and magnetic plasma frequencies increases. Besides this, we have also found the complete band gap in such structure with a glass substrate.

(Received June 01, 2010; accepted July 14, 2010)

Keywords: Dispersion relation, Complete band gap, Photonic crystals, Negative index material

1. Introduction

Photonic crystals fully employ to open a new area for the development of all optical integrated circuits. The idea of the photonic crystals can be traced back to early works done by John [1] and Yablonovitch [2] in 1987. The simplest possible photonic crystal is one dimensional photonic crystal which consists a period of alternate layer of dielectric materials in a direction. The one dimensional photonic crystal with quarter-wave stack acts as a perfect mirror in which light of the proper wavelength is completely reflected [3].

A medium whose dielectric permittivity and magnetic permeability are negative at a given frequency of electromagnetic wave is called a negative index material (NIM) or meta-material. The phase and group velocities of an electro-magnetic wave in such materials can propagate in opposite directions such that the direction of propagation is reversed with respect to the direction of energy flow [4]. This phenomenon is called the negative refractive index and first time it was theoretically proposed by Veselago in 1968 [5].

The photonic band gap corresponding to zero (volume) averaged refractive index is studied in the layered heterostructure containing the ordinary and negative refractive index materials [6]. This new band gap is invariant upon a change of scale length and insensitive to disorder. When an impurity is introduced in such photonic crystal, the defect mode is appeared inside the zero-*n* gap which is very weakly dependent on incident angle. The zero-*n* gap of the structure can also exhibit a photonic band gap which does not based on interference mechanisms usually obtained in the Bragg gap [7-9]. Chen et al. [10] have derived the expression for the incident angle dependence of the frequency of defect modes inside the zero-*n* gap of the one-dimensional photonic crystals stacking with positive and negative index materials. The expression has shown the dependence of the dispersion of the defect mode on both the refractive index and the wave impedance of component layers. The interaction between the zero averaged refractive index, the zero permeability and the zero permittivity gaps are focused and analyzed by the unavoidable dispersive character of meta-materials. The degree of overlap between these bands can be varied by a proper selection of the constructive parameters [11].

The one-dimensional photonic crystals and quasi-periodic structures containing a positive and a negative index material are physically fabricated. The transmittance of the structures has demonstrated experimentally that the zero-*n* gaps is independent on the scaling of the length and is confirmed the crucial function of ε=0 and μ=0 by constructing quasi-periodic structures [12,13]. The dispersion relation and associated electric fields of the one-dimensional photonic crystals composed of alternating layers of right-handed and left-handed materials (RHM and LHM) for the oblique propagation have been investigated. The parameters of dielectric permittivity and magnetic permeability of the RHM are taken constant, whereas both parameters of the LHM are assumed negative simultaneously. The calculation of the dispersion relation and associated electric field are performed on the basis of the plasma frequencies [14].

The complete photonic band gaps for all polarizations have obtained in the periodic structure containing dielectric and meta-material. The origin of the complete PBGs is laid in the existence of surface waves for all polarizations [15]. The modulation of the refractive index in all three spatial dimensions is required to obtain the complete band gap and it prevented the propagation of electromagnetic waves in all directions. The one-dimensional periodic structure containing the layers of transparent left-handed (or negative-index) material can have trapped light in three-dimensional space due to the existence of a complete band gap [16]. Bloemer et al. [15] have studied omni-directional reflection properties in the single layer of a negative index material (NIM). The single negative index material has reflected the radiation for all angles of incidence and both polarizations. The larger reflection bandwidth is obtained by increasing the separation between the electric and magnetic plasma frequencies.

The complete band gap in one-dimensional photonic crystal (1D-PC) containing negative-index material (NIM) is investigated when separation between electric and magnetic plasma frequencies of NIM increases. The separation between electric and magnetic plasma frequencies of NIM affect to the band gap and the transmittance as well. The average zero-*n* gap observes insensitive to incident angles and polarizations. Other hand the Bragg-gap shifts towards the higher frequency with increasing angle of incidence. The transmittance of the structure for TE and TM-modes shows the complete band gap owing to the mechanism of the average zero-*n* when the separation between electric and magnetic plasma frequencies increases. Besides this we have also found the complete band gap for structure with a glass substrate.

2. Theory

The dispersion relation and transmittance of the binary one-dimensional photonic crystal with alternating layers (ε_1, μ_1), and (ε_2, μ_2) with thickness of the layers d_1, and d_2 respectively are shown in the Fig. 1.

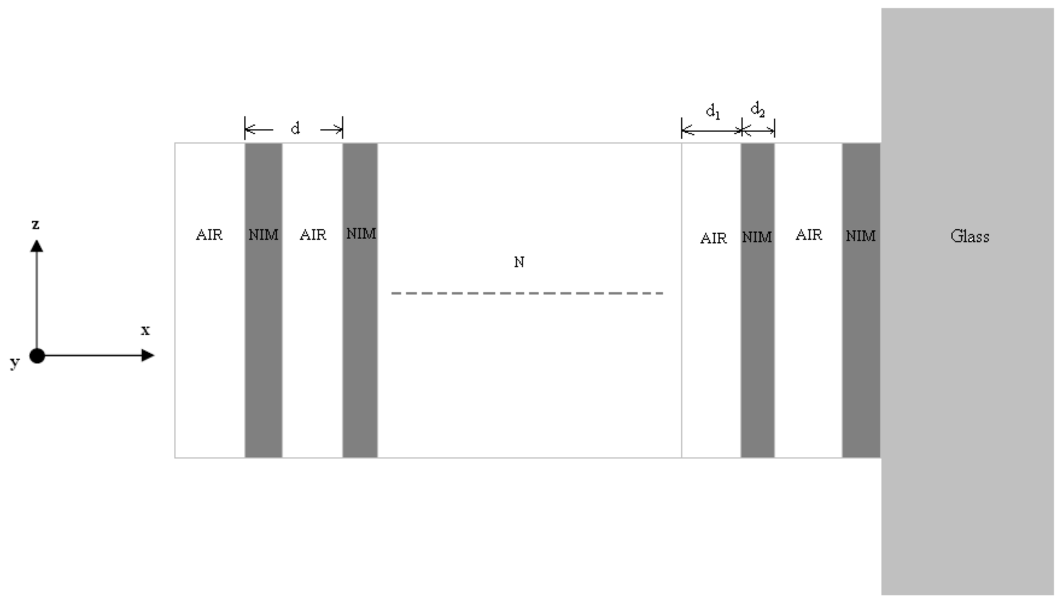


Fig. 1. Schematic diagram of the periodic structure of positive index material (air) and negative index material.

The d (= $d_1 + d_2$) is the periodic of the unit cell. The periodic structure of (ε_1, μ_1), and (ε_2, μ_2) is periodic dependent permittivity $\varepsilon(x + d) = \varepsilon(x)$ and permeability $\mu(x + d) = \mu(x)$. We consider an oblique propagation of monochromatic electromagnetic field (with time dependence $e^{-i\omega t}$) in a periodic structure with oblique wave vector β along the y-axis [6]. For the E-polarization case,

$$E_{1z} = e^{i\beta y}[A e^{ik_{1x}x} + B e^{-ik_{1x}x}],$$

$$H_{1y} = -\frac{k_{1x}}{\omega\mu_1} e^{i\beta y}[Ae^{ik_{1x}x} - Be^{-ik_{1x}x}]$$

$$H_{1x} = -\frac{\beta}{\omega\mu_1} e^{i\beta y}[Ae^{ik_{1x}x} + Be^{-ik_{1x}x}] \text{ in region I} \tag{1}$$

and

$$E_{2z} = e^{i\beta y}[Ce^{ik_{2x}x} + De^{-ik_{2x}x}]; \quad H_{2y} = -\frac{k_{2x}}{\omega\mu_2} e^{i\beta y}[Ce^{ik_{2x}x} - De^{-ik_{2x}x}]$$

$$H_{2x} = -\frac{\beta}{\omega\mu_2} e^{i\beta y}[Ae^{ik_{2x}x} + Be^{-ik_{2x}x}] \quad \text{in region II} \tag{2}$$

where k_{jx} is the component of the wave vector along x-axis in region j=1,2 i.e. $k_j^2 = \varepsilon_j \mu_j \frac{\omega^2}{c^2} - \beta^2$. Here c is the speed of light in vacuum and β is the z-component wave vector. The periodicity of the structure is imposed i.e. E[(x+d),y]=E(x,0)exp(iKd+iβy), where $d=d_1+d_2$ and K is the dimensionless Bloch wave vector of the periodic unit cell [6,16]. The tangential electric and magnetic fields should be continuous at x = 0 and x = d_2 i.e.

$$\left.\begin{aligned} E_{1z}(x=0^-) &= E_{2z}(x=0^+) \\ H_{1y}(x=0^-) &= H_{2y}(x=0^+) \end{aligned}\right\} \tag{3}$$

and

$$\left.\begin{aligned} E_{2z}(x=d_2^-) &= E_{1z}(x=d_2^+) \\ H_{2y}(x=d_2^-) &= H_{1y}(x=d_2^+) \end{aligned}\right\}$$

$$E_{2z}(x=d_2) = E_{1z}(x=-d_1)\,e^{iKd}$$
$$H_{2y}(x=d_2) = H_{1y}(x=-d_1)\,e^{iKd} \tag{4}$$

To obtain the dispersion relation of the one-dimensional photonic crystal containing NIM, we applied the periodic conditions and the Bloch theorem, where K is the first Brillouin zone $-\pi/d \le K \le \pi/d$. The equations (3) and (4) are substituted into equations (1) and (2); we obtained the matrix M of the periodic structure which is characterized the wave scattering;

$$\begin{pmatrix} A_{n-1} \\ B_{n-1} \end{pmatrix} = M \begin{pmatrix} A_n \\ B_n \end{pmatrix} \tag{5}$$

where M is the 2×2 matrix with elements with $M_{11} = e^{ik_{1x}d_1}[\cos(k_{2x}d_2) + \frac{i}{2}(\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1})\sin(k_{2x}d_2)]$, $M_{12} = e^{-ik_{1x}d_1}[\frac{i}{2}(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_2})\sin(k_{2x}d_2)]$, $M_{21} = e^{ik_{1x}d_1}[-\frac{i}{2}(\frac{Z_2}{Z_1} - \frac{Z_1}{Z_2})\sin(k_{2x}d_2)]$ and $M_{22} = e^{-ik_{1x}d_1}[\cos(k_{2x}d_2) - \frac{i}{2}(\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1})\sin(k_{2x}d_2)]$

Then the dispersion relation of the binary structure unit cell (ABAB ... stack) is given as,

$$\cos(Kd) = \frac{1}{2}\mathrm{Tr}[M(\omega)] \tag{6}$$

or

$$\cos(Kd) = \cos(k_{1x}d_1)\cos(k_{2x}d_2) - \frac{1}{2}\left[\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1}\right]\sin(k_{1x}d_1)\sin(k_{2x}d_2) \tag{7}$$

The transmission (t) and reflection (r) coefficients of the structure can be easily related between the plane wave amplitudes in the left, right and back is as follow;

$$\boldsymbol{M(Nd) = [(M(d)]^N} = \begin{pmatrix} \boldsymbol{M}_{11} & \boldsymbol{M}_{12} \\ \boldsymbol{M}_{21} & \boldsymbol{M}_{22} \end{pmatrix} \tag{8}$$

where

$\boldsymbol{M}_{11} = M_{11}U_{N-1} - U_{N-2}$, $\boldsymbol{M}_{12} = M_{12}U_{N-1}$, $\boldsymbol{M}_{22} = M_{22}U_{N-1} - U_{N-2}$, $\boldsymbol{M}_{21} = M_{21}U_{N-1}$ and $U_N = \sin[(N+1)K(\omega)d]/\sin[K(\omega)d]$. So, that the reflection (r) and transmission (t) coefficients are given by;

$$\boldsymbol{r} = \frac{(\boldsymbol{M}_{11} + \boldsymbol{M}_{12}\boldsymbol{p}_s)\boldsymbol{p}_0 - (\boldsymbol{M}_{21} + \boldsymbol{M}_{22}\boldsymbol{p}_s)}{(\boldsymbol{M}_{11} + \boldsymbol{M}_{12}\boldsymbol{p}_s)\boldsymbol{p}_0 + (\boldsymbol{M}_{21} + \boldsymbol{M}_{22}\boldsymbol{p}_s)}, \quad \boldsymbol{t} = \frac{2\boldsymbol{p}_0}{(\boldsymbol{M}_{11} + \boldsymbol{M}_{12}\boldsymbol{p}_s)\boldsymbol{p}_0 + (\boldsymbol{M}_{21} + \boldsymbol{M}_{22}\boldsymbol{p}_s)} \tag{9}$$

Here p_0 and p_s are the substrate of first and last medium and given as; $\boldsymbol{p}_0 = \frac{\boldsymbol{n}_0\cos\boldsymbol{\theta}_0}{\boldsymbol{Z}_0}$, $\boldsymbol{p}_s = \frac{\boldsymbol{n}_s\cos\boldsymbol{\theta}_s}{\boldsymbol{Z}_0}$ $(\boldsymbol{TE})$ and $\boldsymbol{p}_0 = \frac{\cos\boldsymbol{\theta}_0}{\boldsymbol{Z}_0\boldsymbol{n}_0}$, $\boldsymbol{p}_s = \frac{\cos\boldsymbol{\theta}_s}{\boldsymbol{Z}_0\boldsymbol{n}_s}$ $(\boldsymbol{TM})$

The reflectance (R) and transmittance (T) for considered structure are related by:

$$\boldsymbol{R} = |\boldsymbol{r}|^2 \quad \textbf{and} \quad \boldsymbol{T} = \frac{\boldsymbol{p}_s}{\boldsymbol{p}_0}|\boldsymbol{t}|^2 \tag{10}$$

3. Results and discussion

For numerical analysis, we have taken a periodic structure of the air and negative index material. The substrates of the structure are considered the air (n=1.0) and glass (n=1.5). The effective permittivity $\varepsilon(\omega)$ and permeability $\mu(\omega)$ of the NIM material is considered by [6];

$$\varepsilon(\omega) = 1 + \frac{5^2}{0\cdot 9^2 - \omega^2} + \frac{10^2}{11\cdot 5^2 - \omega^2} \tag{11}$$

$$\mu(\omega) = 1 + \frac{3^2}{0\cdot 902^2 - \omega^2} \tag{12}$$

We have calculated the dispersion and transmittance of the structure for both TE and TM-modes at different angles of incidences using equations (7) and (10) with different substrates. The separation between the electric and magnetic plasma frequencies of NIM is increased to study the optical properties of the structure like band structure and transmittance. Fig. 2 shows the electric permittivity, magnetic permeability and the optical constant (n) of the NIM versus frequency (GHz). The refractive index of NIM is found negative due to the effective parameters of the ε and μ are negative.

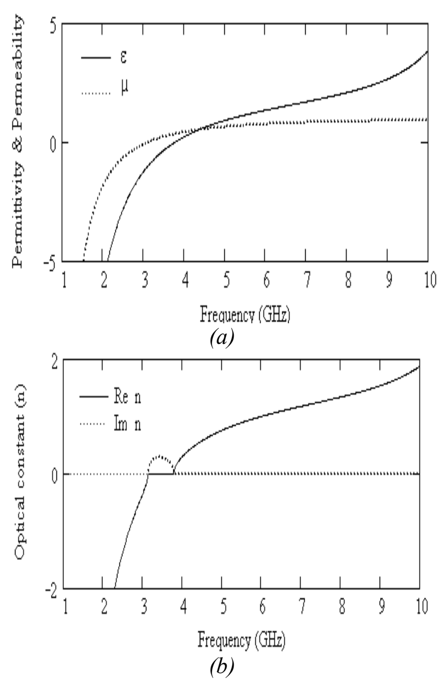
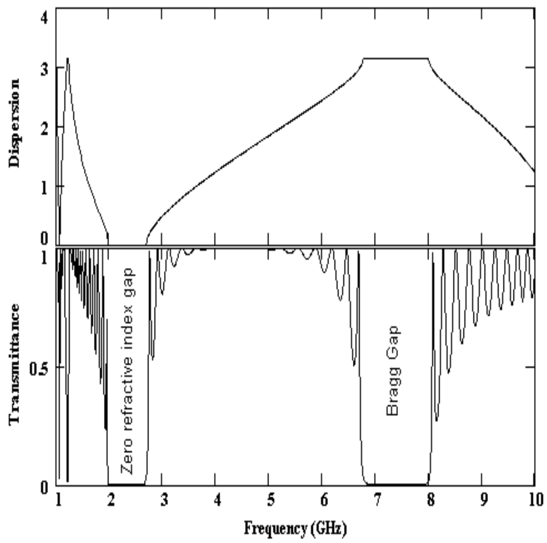
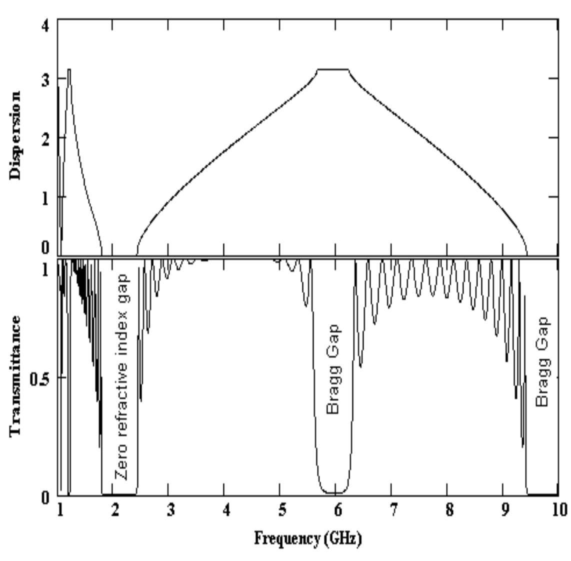


Fig. 2. (a) Electric permittivity and magnetic permeability of NIM (b) optical constant of the NIM.

Using these parameters, we have calculated the dispersion and transmittance of the binary periodic structure containing air/NIM materials and glass/NIM materials with air substrate, as shown in the Fig. 3. The band structures and the transmittance of the structure have obtained two types of the band gaps: *zero-gap and Bragg-gap*.



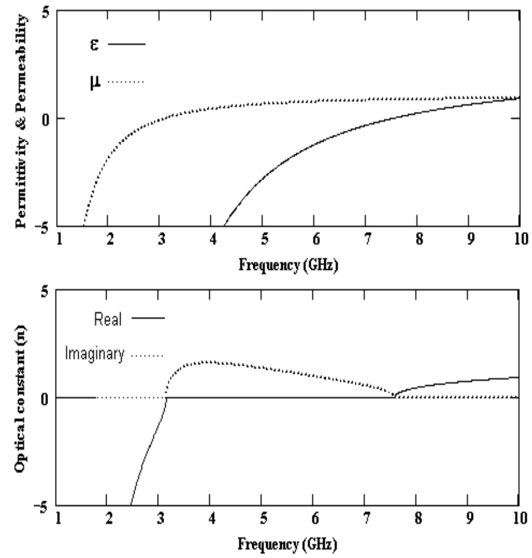
(a)



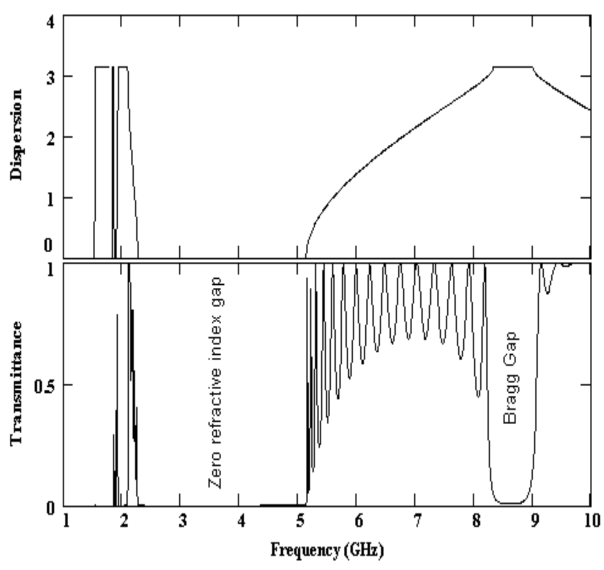
(b)

Fig. 3. Dispersion relation and transmittance of the binary structure of (a) air/NIM materials and (b) glass/NIM materials with air substrate.

The band structure and transmittance of the periodic structure are calculated by changing following cases: (i) electric plasma frequency, (ii) magnetic plasma frequency and (iii) both electric and magnetic plasma frequencies of the NIM in the equations (11) and (12) which is already discussed by Bloemer et al. [17] for single layer of the NIM. The Fig. 4 (a) shows optical constants of the NIM when the electric plasma frequency is double of the initial value and magnetic plasma frequency constant. The interesting results of transmittance spectra are found: the width of zero-gap is increased and Bragg gap is shifted towards higher frequencies as shown in the Fig. 4 (b).



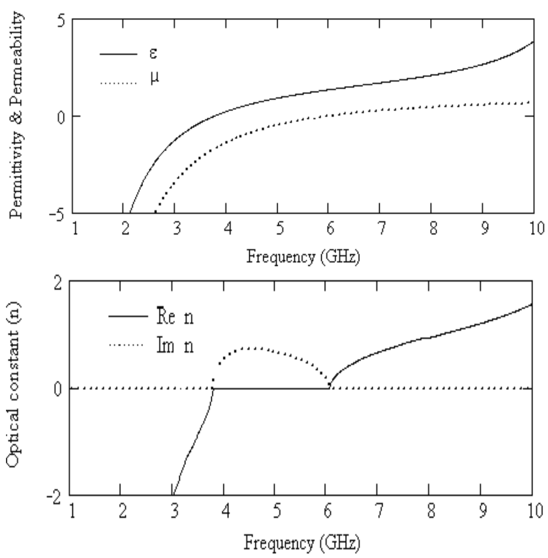
(a)



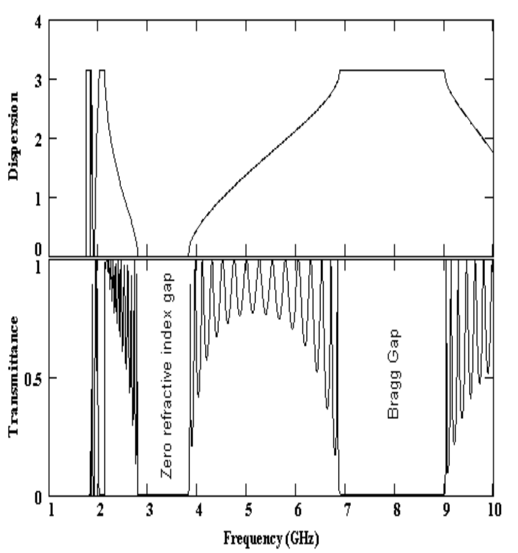
(b)

Fig. 4. (a) Electric permittivity, magnetic permeability of NIM and optical constant of the NIM (b) Dispersion relation and transmittance of the binary structure of air/NIM materials with twice electric plasma frequency.

Fig. 5 (a) shows optical constants of NIM when magnetic plasma frequency is increased two times of the initial value and electric plasma frequency remains constant. The band structure and transmittance spectra for this condition are shown in Fig. 5 (b). The zero-gap of the considered periodic structure is increased compare to conventional photonic crystals with NIM. The Figs. 4 (b) and 5 (b) are predicted that the electric plasma frequency is more effective to enhance the band gap and transmittance. The larger zero refractive index is obtained by increasing the separation between the electric and magnetic plasma frequencies. The zero refractive index of the material has found large range of the frequency when the electric plasma frequency is doubled. The dispersion and transmittance of the structure show the zero-gap and the Bragg-gap at the lower and the higher frequency ranges, respectively.



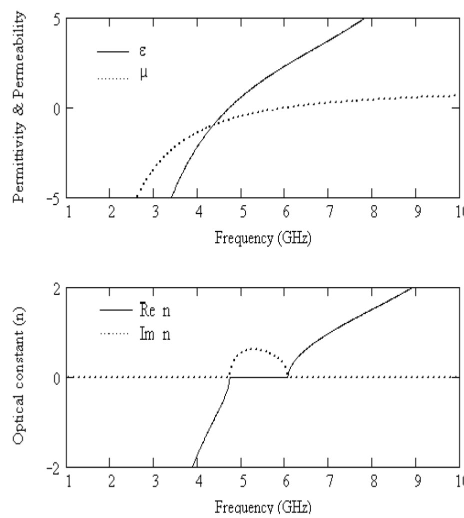
(a)



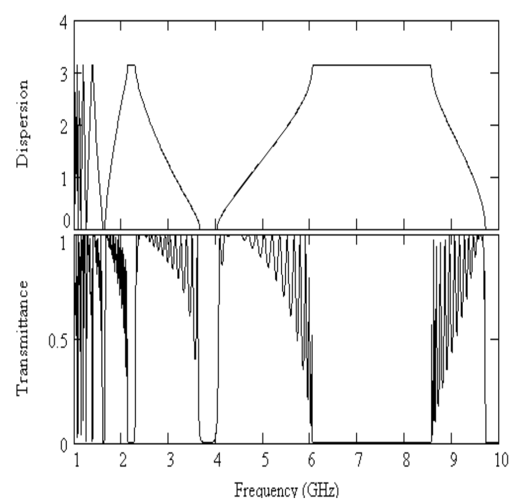
(b)

Fig. 5. (a) Electric permittivity, magnetic permeability of NIM and optical constant of the NIM (b) Dispersion relation and transmittance of the binary structure of air/NIM materials with twice magnetic plasma frequency.

Similarly, we have increased the magnetic and the electric plasma frequencies of the NIM, the optical properties of the structure are shown in the Fig. 6. The increased plasma frequency of the electric and magnetic plasma frequencies in the NIM affect the zero-gap as well as Bragg-gaps of the structure. The size of the zero-gap is found to decrease but increases the number of the zero-gaps and other hand the Bragg-gap of the structure broadens at higher frequency range. By analyzing the Figs. 4, 5 and 6 of the zero-n gaps, we concluded that the zero-n gap is found large when the electric plasma frequency is increased. So it may be considered that the electric plasma frequency of the NIM is more effective in the multiple reflections at the interfaces of the binary periodic structure containing NIM.



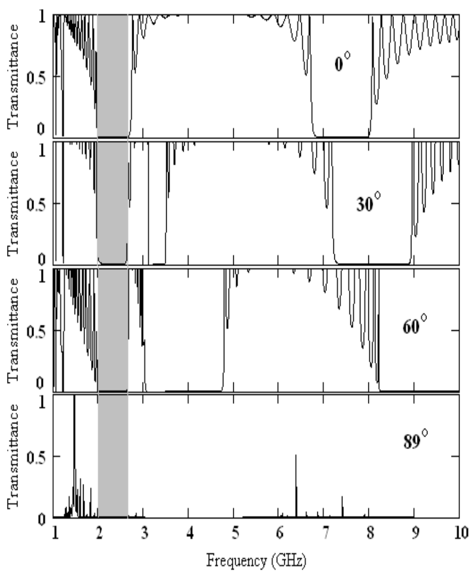
(a)



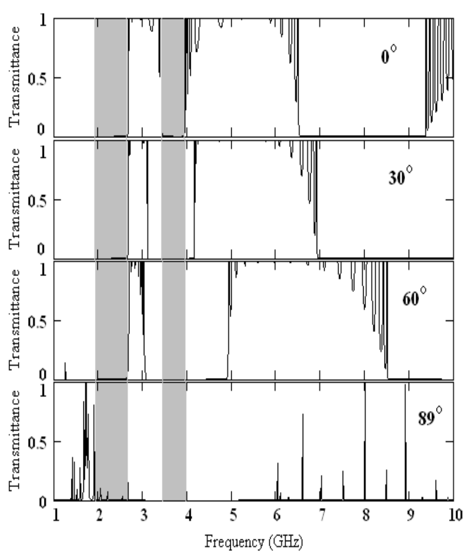
(b)

Fig. 6. (a) Electric permittivity, magnetic permeability of NIM and optical constant of the NIM (b) Dispersion relation and transmittance of the binary structure of air/NIM materials with twice electric and magnetic plasma frequencies.

The transmittance of the considered structure are calculated for different angle of incidences like 0^o, 30^o, 60^o and 89^o for TE and TM modes as shown in Fig. 7. The band gap and transmittance of the considered structure are not shifted for the different angle of incidences at the low frequency range. This gap shows a new type of gap which is different from the Bragg gap. This average zero-gap is obtained due to the presence of the zero and negative refractive index inside the structure. On other hand the band structure and transmittance of the binary structure of the air/NIM for different angle of incidences is shifted towards the higher frequency.



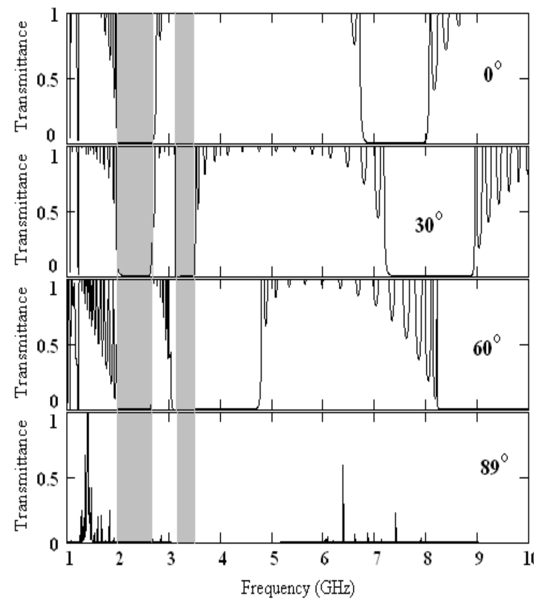
(a) TE mode



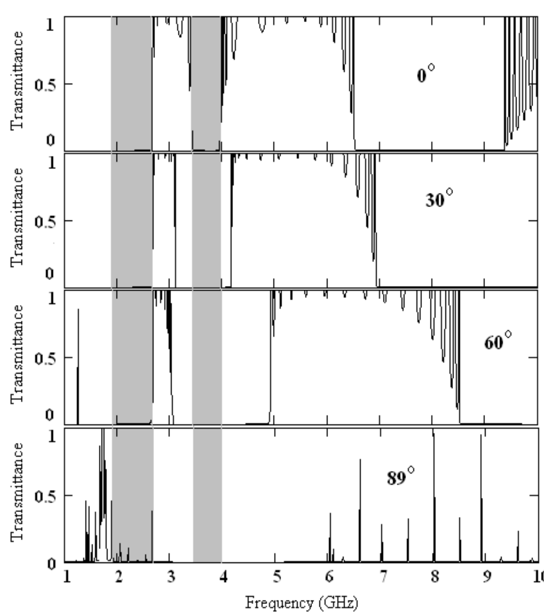
(b) TM mode

Fig. 7. Transmittance of the binary structure of air/NIM materials with air substrate at different angle of incidence (a) TE and (b) TM-modes.

The Bragg-gaps are generally depending upon the angle of incidences. Similar calculation has been done for the periodic structure of air/NIM with glass substrate (n=1.5) for TE and TM modes which is shown in the Fig. 8. The transmittance of the structure at different angle of incidences is as predicted a common band in the lower frequency range of the spectrum. This observation shows the complete band gap of the binary structure of the air/NIM materials.



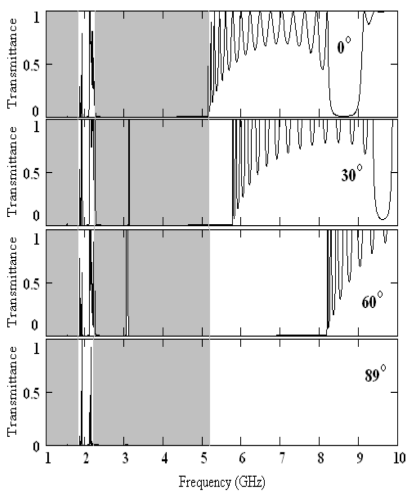
(a) TE mode



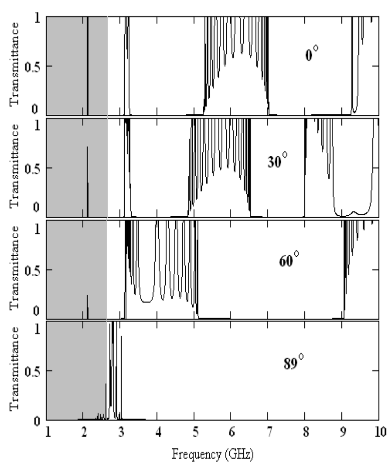
(b) TM mode

Fig. 8. Transmittance of the binary structure of air/NIM materials with glass substrate at different angle of incidence (a) TE and (b) TM-modes.

Now we have taken the NIM with twice time of the electric plasma frequency to calculate the transmittance for TE and TM modes at the different angle of incidences as shown in the Fig. 9. The transmittance of the structure for TE and TM-modes at 0^o, 30^o, 60^o and 89^o is shown the omni-directional behavior for both polarizations. In the Fig. 9 (a), the transmittance spectra for TE mode of the air/NIM materials with air substrate shows that the width of the zero gaps remains invariant at different angles of incidence. The transmittance for TE mode is obtained for the spectral ranges 0.0GHz-1.7GHz and 2.2GHz-5.1GHz. The Fig. 9 (b) shows the transmittance spectra for TM mode at different angles 0^o, 30^o, 60^o and 89^o, we have seen that the large zero-gap obtain with all angles of incidence due to existence of $\mu=0$ and $\varepsilon\neq0$. The transmittance for TE and TM-modes shows a complete band gap owing to the mechanism of the average zero-*n* when the separation between the electric and the magnetic plasma frequencies increases. Besides this we have also found the complete band gap with a glass substrate.



(a) TE mode



(b) TM mode

Fig. 9. Transmittance of the binary structure with twice electric plasma frequency of air/NIM materials at different angle of incidence (a) TE and (b) TM-modes.

4. Conclusions

The complete band gap in the one-dimensional photonic crystal (1D-PC) containing a negative-index material (NIM) are obtained when separation between electric and magnetic plasma frequencies of NIM increases. The 1D-PC has found two types of band gap: the average zero-*n* and the Bragg gap. The feature of the average zero-gap of the 1D-PC containing NIM is the angular independence but it is totally dependent of the separation between the electric plasma and magnetic plasma frequencies. The Bragg-gap is shifted towards the higher frequency with increasing angle due to the change in the optical path through each layer as the refraction angle changes. The average zero-gap of the structure does not depend upon angle and thicknesses but actually widens with increasing the electric plasma frequency. The transmittance of the 1D-PC containing NIM with glass substrate has also obtained two common band gaps due to the refractive index of glass. The transmittance of the structure for TE and TM-modes shows a complete band gap owing to the mechanism of the average zero-*n* when the separation between electric and magnetic plasma frequencies increases.

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